NovAtel Inc. New Positioning Filter: Phase Smoothing in the Position Domain

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BIOGRAPHIES

Tom Ford is a GPS specialist at NovAtel Inc.. He has a BMath degree from the University of Waterloo (1975) and a BSc in survey science from the University of Toronto (1981). He became involved with inertial and GPS technologies at Sheltech and Nortech surveys in 1981. He is a member of the original group of GPS receiver developers at NovAtel Inc., where he has helped develop many of the core tracking, positioning and attitude determination technologies used there. His current focus is the integration of GPS other supplementary systems, especially INS.

Jason Hamilton has been a geomatics EIT at NovAtel Inc. since he graduated with a BSc in Geomatics Engineering from the University of Calgary in 1998. He has worked in the test and OEM development group until 2001, when he became a member of the research team at NovAtel Inc. He is focusing his efforts on GPS/INS integration and the use of phase measurements to enhance the GPS position.

ABSTRACT

During the early days of GPS navigation, Ron Hatch at Magnavox designed a filter that combined delta phase measurements and pseudoranges into a single noise reduced measurement. While reducing the noise on the measurement used in the navigation solution, the reduction of the effect of multipath was not as much as was hoped because of the biased nature of the multipath signal on the pseudorange. At the same time, the time constant in the filter had to be limited because the ionospheric phase advance was a different sign than the pseudorange ionospheric group delay error. Finally, the effectiveness of the phase smoothing technique was limited because in a kinematic environment, frequent signal outages occur and every time this happened all of the smoothed pseudorange information was lost and the accuracy of the pseudorange reverted back to its nominal unsmoothed level.

In a differential system position, precision can be improved through the use of the phase ambiguity, which allows the receiver to treat the carrier measurement as a range with ever better precision. This can eventually provide a receiver with a position accuracy at the centimetre level, provided the differential base station is close enough to the receiver. If satellites tracking is interrupted, the position precision in particular directions (depending on the locations of the satellites) can be maintained via the ambiguities of the satellites the system continues to track.

In a non-differential system, delta phase measurements in conjunction with a velocity model can be used to help estimate average velocity that can help maintain position accuracy when the constellation drops below 4 satellites, and to help reduce the effect of pseudorange errors when the number of satellites is 4 or more. But the delta phase measurement only measures average velocity, so some assumptions about the system dynamics has to be made and this adds the requirement of additional system noise in the positioning filter which reduces its accuracy.

This paper describes a method for combining the delta phase measurement in a filter which includes the current and the previous position. With both the current and previous position in the filter, a position difference can be formed which is directly observable by the phase difference measured between the previous and current time epochs. The previous and current position difference is completely observable by four phase differences or partially observable if less than three satellites are continuously available. The advantage of this method over phase smoothing is that in order for the filter to make use of the delta phase measurement, it only needs to be available since the previous time epoch, rather than over the last 50 seconds or so. Provided that some selection of 4 satellites are available over every epoch, the position accuracy of the system can be

maintained and improved. This is in contrast to the phase smoothing technique in which the same four satellites must be continuously tracked in order for the position accuracy to be maintained and improved by the same amount. The advantage of this method over a differential process is that it does not need any base station infrastructure and is simpler than typical RTK algorithms. The accuracy of the system is at the 1 to 2 metre level when the geometry is good and at the 5 to 10 metre level in urban canyons. In the same urban canyon environment, a pseudorange only solution using a least squares technique, the accuracy often degrades to the 100 metre level, so this approach shows a vast improvement over conventional methods.

Over the past year, NovAtel Inc. has developed a new filter which uses delta phase measurements as inputs to a filter that maintains both the current and previous position to drastically improve the positioning accuracy in areas where the sky is partially or intermittently obscured. In this paper the positioning algorithm is described and test results showing the positioning improvement over conventional least squares is presented. Results from both inner city (old growth forest) residential neighbourhoods and urban canyons are shown.

INTRODUCTION

An observation equation links pseudorange and position via the geometrical information provided by the satellite in its orbital data. A large portion of the change in pseudorange related to the geometrical change in the satellite's position is largely represented by the change in the carrier measurement from the satellite to the receiver, provided the carrier signal has been continuously tracked at the receiver. Combining these can reduce the noise on the pseudorange measurement significantly, so at first glance, the combination of pseudorange and carrier measurements to generate an enhanced pseudorange measurement is an attractive method. However, the refined measurement has three conditions which limit its usefulness. First, the ionospheric phase advance is equal and opposite to the ionospheric group delay, so over time the change in pseudorange deviates from the change in carrier according to the ionospheric change. Secondly, pseudorange errors corrupted by multipath are biased [6], so the combined pseudorange carrier measurement error is difficult to estimate as it is a function of the multipath environment. Finally, the noise on the refined pseudorange takes time to be reduced. A reduction of the noise by a factor of 10 will take 100 seconds of continuous phase tracking, and often carrier tracking is interrupted on individual satellites more frequently than this, so the carrier smoothed pseudorange never reaches steady state.

In some environments, various satellites are obstructed periodically. In some cases, the minimum number of satellites may be available for a solution all the time, but it is possible that the tracking duration for all the satellites is short. In this environment, carrier smoothing the pseudoranges does not help much because none of the individual satellites are tracked long enough to reduce the variance for the carrier smoothed observations. Intuitively, enough information should be available from delta carrier measurements so that the epoch to epoch position change should be determined to the level of the delta carrier accuracy, provided at least 4 delta carrier measurements are available. It is in fact possible to account for all the vehicle dynamics with delta carrier measurements in a least squares approach [5]. In this method, both the current and previous positions are included as variables in a least squares adjustment. The idea in this paper is to use the delta carrier measurements as observables in a Kalman filter which incorporates the current position, velocity and possibly clock as well as the previous position.

The motivation for the new filter approach came from Sportvision, a customer of NovAtel Inc. They wanted to have meter level positioning accuracy (2 sigma) on NASCAR race cars so they could provide real time computer graphics that followed the racecars as they went across the television screen. The navigation difficulty in this problem was that better than normal pseudorange positioning was required, but the duration of the satellite constellation was too short for either fixed ambiguity positioning or accurate floating ambiguity positioning. Although the incorporation of track model data into the position solution [6][7] satisfied (to paraphrase Lincoln), the positioning requirements on some of the tracks all of the time and all of the tracks some of the time, it couldn't satisfy the positioning requirements on all of the tracks all of the time. The Kalman filter approach, with current position, velocity and clock states, as well as the previous position state with differential pseudorange and delta carrier measurements as observations satisfied the requirements to the extent that the technology is used during nearly every race.

During analysis of data collected in the urban canyons in downtown Calgary, it became evident that position errors from a filter that included clock and clock rate estimates would be adversely affected by clock and clock rate errors when the system did not have enough observations to generate an instantaneous position and clock estimate. As a result, the filter was modified so that clock and clock rate parameters were not estimated. Instead, pseudorange, doppler and delta phase measurements were all differenced across satellites before they were used in the Kalman filter to help estimate position and velocity

KALMAN FILTER FORMULATION

The Kalman filter is well documented in references like [1][2][3] consists of a propagation step and an update step. The Kalman propagation reflects the effect of dynamics over time on the state and of dynamics and time related uncertainties on the state covariance. The update functions to combine information in the state and its covariance with that of external observations and their covariance, provided some functional relationship exists between the state and the observations. The Kalman filter equations are copied out for reference, along with the specific definitions of the Kalman elements to satisfy position and velocity estimation from GPS observations. Then the filter element modifications are described which incorporate the delta phase measurements into the filter.

KALMAN EQUATIONS

The specific Kalman filter definition varies with the implementation. The specification of 7 basic elements define the filter to the extent that it can be implemented.

1) x: State vector
2) P: State Covariance matrix
3) Φ: Transition matrix (differential equation solution)
4) Q: Process noise matrix (effect of incorrect modelling over time)
5) z: Measurement vector
6) R: Measurement Covariance matrix
7) H: Linear Relationship of measurement to state

Following [1] or [2], the Kalman filter mechanisation can be specified as a sequence of state and covariance propagation steps followed by one or more update steps.

Propagation step:
State propagation: $x_t(-) = \Phi x_{t-1}(+)$
Covariance propagation: $P_t(-) = \Phi P_{t-1}(+)\Phi^T + Q$

Update Step:
Gain computation: $K = P(-)H^T(HPH^T + R)^{-1}$
State Update: $x(+) = x(-) + K(z - Hx(-))$
Covariance Update: $P(+) = (I - KH)\, P(-)$

If a position/velocity filter is to be used, the state vector will have 6 elements. The reference frame used for the computation will be the ECEF frame, so the state elements will be:

State: $x=[\delta x, \delta y, \delta z, \delta v_x, \delta v_y, \delta v_z]$
The elements are preceded by the δ symbol to indicate they are error states, not system elements.

The covariance matrix associated with the pseudorange/delta phase (PDP) implementation is initialised as diagonal 6 by 6 matrix with large diagonal elements. The seed position for the system will be provided by the least squares process, so the position error states can be assumed to have an initial variance of (100 metres)^2, and the velocity error states can be assumed have an initial variance of (100 metres/sec)^2.

State Initial Covariance: P= [big diagonal elements, 0 off diagonal elements]

This particular filter maintains only position and velocity states. In order for the clock components of the system to be eliminated, all pseudorange observations are single differenced (across satellites) to eliminate the common clock offset. All doppler and delta phase measurements are also differenced to eliminate clock rate. The Kalman propagation is dependent on the solution of the differential equations describing the dynamics of the state elements. This contains both deterministic and stochastic portions. Since only position and velocity elements are estimated, the following dynamics matrix described the state error growth under assumed constant velocity conditions.

$$d\dot{x} = Fx + w$$

$$F = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

That is, F is a 6 by 6 dynamics matrix with constant coefficients and w is a vector of white noise forcing functions.

Since the F matrix has constant coefficients, the differential equation solution can written as $\Phi(\Delta t) = e^{F\Delta t}$
and for the F matrix in the random walk process seen below, this becomes $\Phi(\Delta t) = I + F\Delta t$ or:

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The solution of the deterministic portion provides a transition matrix, and the solution of the stochastic portion provides a Q matrix. The process noise matrix Q is based on the transition and the spectral densities Q(τ) of the random forcing functions associated with the state according to the equation (following [8] for example).

$$Q_{ECEF} = \int_0^{\Delta t} \Phi(t)Q(t)\Phi(t)^T dt$$

Where Q(τ) is a spectral density matrix for the random forcing function vector for the state elements. In general the spectral densities for the state element forcing functions are not known, and for this filter these will vary with the system dynamics. So the spectral densities for the position and velocity will be chosen heuristically such that the propagated covariance reflects the actual performance of the system. If the theoretical advantage of a local level spectral density formulation is ignored, the Q_{ECEF} derivation is simple and an analytic expression can be generated because the quantity $Q(\tau)_{VEL\_ECEF}$ is not position dependent. In this case, the $Q(\tau)_{diag}$ is given by:

$Q(\tau)_{diag} = (q_p, q_p, q_p, q_v, q_v, q_v)$

With q_v being the common spectral density for all the velocity elements.

Then the Q_{ECEF} matrix is zero except for the following elements:

$$Q_{11} = Q_{22} = Q_{33} = q_p\Delta t + q_v\Delta t^3/3$$
$$Q_{44} = Q_{55} = Q_{66} = q_v\Delta t$$
$$Q_{14} = Q_{41} = Q_{25} = Q_{52} = Q_{36} = Q_{63} = q_v\Delta t^2/2$$

Only the non-zero computed elements are applied to the P matrix elements. The spectral density for the velocity is derived from the cleaned doppler misclosures, so the filter is automatically adaptive to changes in system dynamics. Similarly, the spectral densities for position are derived from the delta phase innovations.

KALMAN UPDATE

The linear relationship between the measurements and the state are derived as a matrix of partial derivatives of the functions which link the measurements and the state elements. If such functions don't exist, then the state is not observable with the measurement set. Once the linear relationship "H" between the state and the measurement set is determined, the update process follows the update step describe earlier.

Finally, pseudorange and doppler measurements can be used to estimate the state elements. The description of the pertinent linear relationships (H matrix) follows, first for pseudorange and position, and then for doppler as it relates to the velocity states.

For the pseudorange difference between satellites i and j and state, a linear relationship can be defined based on the positions of the satellite and the receiver. Assuming the single difference is defined as:

$\Delta\rho^{ij} = \rho^j - \rho^i$

$H = [\Delta x^i/R^i - \Delta x^j/R^j, \Delta y^i/R^i - \Delta y^j/R^j, \Delta z^i/R^i - \Delta z^j/R^j, 0, 0, 0]$

Where
$\Delta x^i = x^i - x_r$, the difference between the x components of the i^{th} satellite and the receiver, with similar expressions for the other difference elements, and
$R^i = ((\Delta x^i)^2 + (\Delta y^i)^2 + (\Delta z^i)^2)^{1/2}$ represents the best estimate of the geometric range to the satellite from the receiver.

The measurement which is most closely related to the position in the filter is the reduced pseudorange, that is, the measured pseudorange minus the theoretical pseudorange. So inherent in this process is the presumption that a "system" is maintained with the help of the Kalman filter which in fact estimates error states or corrections to the system.
In the state update equation using pseudorange differences:
$x(+) = K(z - Hx(-))$, $z = \Delta z_m - \Delta z_s$
Where Δz_m is the measured pseudorange difference and Δz_s is the pseudorange difference reconstructed by the system.

For the reduced doppler difference measurement from satellites i and j, the linear relationship "H" with the velocity state is:
$H = [0,0,0,\Delta x^j/R^j - \Delta x^i/R^i, \Delta y^j/R^j - \Delta y^i/R^i, \Delta z^j/R^j - \Delta z^i/R^i]$

A single reduced doppler measurement is z_{rnd} = Raw Doppler – Satellite clock rate – Satellite motion in the line of sight direction. The observation used in the Kalman filter is just the difference of two different reduced doppler measurements. That is $z = z_{rnd}{}^j - z_{rnd}{}^i$

So now a misclosure or innovation, "w" for the doppler measurement can be defined as

$w = z - Hx(-)$

MODIFICATION TO INCORPORATE DELTA PHASE

The change in phase measurement over time can provide an estimate of the change in the receiver position over time in the direction of the satellite generating the phase. This measurement would be exact except that over time, changes in satellite position, changes in tropospheric and ionospheric delay and changes in the receiver clock all occur. The measurement is also not normally incorporated in a Kalman filter because the Kalman filter states represent system errors at a particular time, while a delta phase or delta position measurement represent an integrated velocity over time. So incorporation of this measurement into the Kalman filter, while attractive, has some difficulties which must be overcome.

The satellite motion can be accounted for based on the user's knowledge of the satellite orbit. The residual error in satellite motion resulting from changes in the satellite position error from ephemeris shortcomings are small compared to the atmospheric error changes. The tropospheric and ionospheric error changes are partly accounted for in the error models associated with the measurements, and partly by the process noise applied to the position in the propagation portion of the Kalman filter. The clock rate component can be eliminated by differencing delta phase measurements across satellites (effectively forming double difference measurements). By using a phase measurement differenced twice across time and satellites, the phase component generated by the change in receiver clock can be eliminated. Based on this, the observation equation relating the phase and delta position is as follows:

The single difference phase across time can be modelled as:
$\Delta\varphi_{t1t2}{}^{j} = H^{j}\ (x_{t1} - x_{t0}) + \Delta\text{Clock}$

Where H is the vector $H^{j} = [-\Delta x^{j}/R^{j}, -\Delta y^{j}/R^{j}, -\Delta z^{j}/R^{j}]$

and $x_{t1} - x_{t0}$ is the vector of position differences between t_1 (the current time) and t_0 (the previous time).
The double difference phase across time and satellites is:

$\nabla\Delta\varphi_{t1t2}{}^{ij} = \Delta\varphi_{t1t2}{}^{j} - \Delta\varphi_{t1t2}{}^{i} = \nabla H^{ij}\ (x_{t2} - x_{t1})$

Where ∇H^{ij} is the vector
$\nabla H^{ij} = [\Delta x^{j}/R^{i} - \Delta x^{j}/R^{j}, \Delta y^{i}/R^{i} - \Delta y^{j}/R^{j}, \Delta z^{i}/R^{i} - \Delta z^{j}/R^{j}]$

The only problem with this formulation is that $\nabla H^{ij}\ (x_{t1} - x_{t0})$, requires that the position at t_1 and the position at t_0 are available. That is, the state must be expanded to include the position at the last epoch.

The state is now defined as $x = [p_1,v,p_0]^T$ where
Current position error vector: $p_1 = [x,y,z]$
Current velocity error vector: $v = [v_x,v_y,v_z]$
Previous position error vector: $p_0 = [x,y,z]$

The Kalman propagation must be modified to not only support the previously defined dynamics equations for the random walk model, but also to transfer the p_1 elements to the p_0 spot in the state vector during the propagation. That is, the current position after the previous update becomes the previous position after the propagation. At the same time, the current position error is propagated according to the estimated velocity error. The modified transition matrix becomes:

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the update can be applied to an extended state for observation $\nabla\Delta\varphi_{t1t2}{}^{ij}$ with an H vector

$H^{ij} = [\Delta x^{i}/R^{i} - \Delta x^{j}/R^{j}, \Delta y^{i}/R^{i} - \Delta y^{j}/R^{j}, \Delta z^{i}/R^{i} - \Delta z^{j}/R^{j}, 0, 0, 0, -\Delta x^{i}/R^{i} + \Delta x^{j}/R^{j}, -\Delta y^{i}/R^{i} + \Delta y^{j}/R^{j}, -\Delta z^{i}/R^{i} + \Delta z^{j}/R^{j}]$

applied in the gain computation

$K = PH^{T}(HPH^{T}+R)^{-1}$

and the reduced double difference phase observable is applied to the state via the following update equation:

$x(+) = x(-) + K\ [\nabla\Delta\varphi_{t1t2}{}^{ij} - H^{ij}\ x(-)]$

Note that x(-) and x(+) are a combination (sum) of state (ie system errors) and system.

DELTA PHASE THEORETICAL EXAMPLE:

It is instructive to look at a simplified propagation and update series for a reduced three state filter representing motion along a single axis. The states consist of the previous and current positions on the axis and the velocity along the axis.

Given the initial state
$x = [p_1,v,p_0]^T$

and associated covariance at time t_1

$$P_0 = \begin{bmatrix} s_{p1}^{2} & 0 & 0 \\ 0 & s_{v}^{2} & 0 \\ 0 & 0 & s_{p0}^{2} \end{bmatrix}$$

The simplified transition matrix will be (substituting t for Δt)

$$\Phi = \begin{bmatrix} 1 & t & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The state propagation gives

$$x(-) = \Phi x(+) = \begin{bmatrix} 1 & t & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ v \\ p_0 \end{bmatrix} = \begin{bmatrix} p_1 + tv \\ v \\ p_1 \end{bmatrix} = \begin{bmatrix} p_2 \\ v \\ p_1 \end{bmatrix}$$

and covariance propagation gives

$$P_t(-) = \Phi \begin{bmatrix} s_{p1}^2 & 0 & 0 \\ 0 & s_v^2 & 0 \\ 0 & 0 & s_{p0}^2 \end{bmatrix} \Phi^T + Q$$
$$= \begin{bmatrix} s_{p1}^2 + t^2 s_v^2 + q_p & ts_v^2 & s_{p1}^2 \\ ts_v^2 & s_v^2 + q_v & 0 \\ s_{p1}^2 & 0 & s_{p1}^2 + q_p \end{bmatrix}$$

This clearly generates a covariance matrix with highly correlated position elements. In fact, the P matrix remains positive definite only because of the uncertainty in the velocity state and the process noise added to the diagonal elements. But a position (or pseudorange) update will affect both the current and to a lesser extent previous position states. Assume the phase measurement geometry is such that all the phase information is in the direction of the modelled axis. Then, the H matrix for the phase observation is H = [1,0,-1]. If a single phase observation with a variance of σ_φ^2 is used in the update, R = σ_φ^2 and an expression for the gain can be written:

$$K = PH^T[HPH^T + R]^{-1}$$
$$= \begin{bmatrix} t^2 s_v^2 + q_p \\ ts_v^2 \\ q_p \end{bmatrix} / (t^2 s_v^2 + q_p + s_j^2)$$

The gain matrix, for a small phase variance will be close to 1.0 for the current position element, close to 1.0/t for the velocity element, and close to 0.0 for the previous position element. If there is an error in velocity, say ε_v, then the error in position will be $\varepsilon_p = t\varepsilon_v$, and this will be reflected in the phase measurement to the extent of the accuracy of the phase observable and the geometry. In this case the geometry is excellent, so the position error is almost entirely represented by the phase measurement (assume a phase noise increment of η_φ). Therefore, during the phase update, the position correction (assuming for simplicity that the previous state vector was zero) will be

$$x(+) = x(-) + K[-te_v + h_j - x(-)] \cong x(-) + \begin{bmatrix} -te_v + h_j \\ (-te_v + h_j)/t \\ 0 \end{bmatrix}$$

Then, the system's current position will be reduced by almost the exact amount ($t\varepsilon_v$) by which it was in error, and the velocity will be reduced by ε_v, the amount by which it was in error. So if the geometry is good, and the error on the phase is small, the relative position and velocity errors will be almost eliminated with the phase update.

The current position uncertainty during the update is modified according to:

$$P(+) = [I - KH]P(-)$$
$$P(+)_{(0,0)} = s_{p1}^2 + t^2 s_v^2 + q_p$$
$$- (t^2 s_v^2 + q_p) \frac{t^2 s_v^2 + q_p}{t^2 s_v^2 + q_p + s_j^2}$$

which for a small phase variance reduces to:

$$P(+)_{(0,0)} = s_{p1}^2$$

eliminating not only the effect of velocity error over time on the current position, but also the effect of system noise on the current position.

Similarly, the effect of reported velocity error is shown to be:

$$P(+)_{(1,1)} = s_v^2 + q_v - s_v^2 \frac{t^2 s_v^2}{t^2 s_v^2 + q_v + s_j^2}$$

which for small phase noise and small velocity system noise, relative to the velocity uncertainty, reduces to:

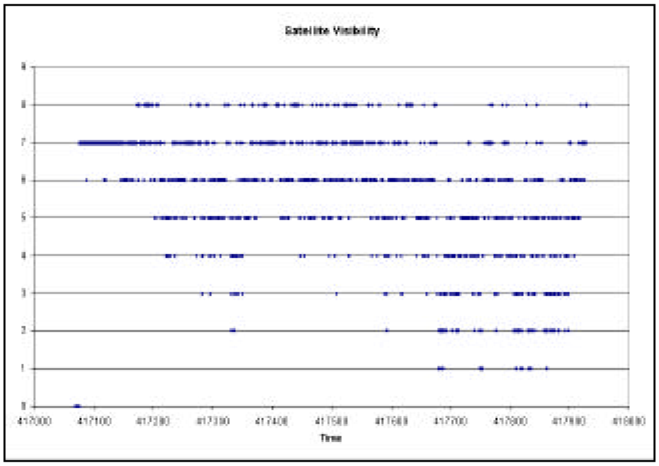
$$P(+)_{(1,1)} = q_v$$

The conclusion to be drawn from this example of a simplified system is that the delta phase measurement can be used with this technique in a Kalman filter to completely compensate for the degradation in knowledge of position from velocity error or any other time related source, provided the phase is accurate enough, and provided the geometry relating phase change to position change is strong enough.

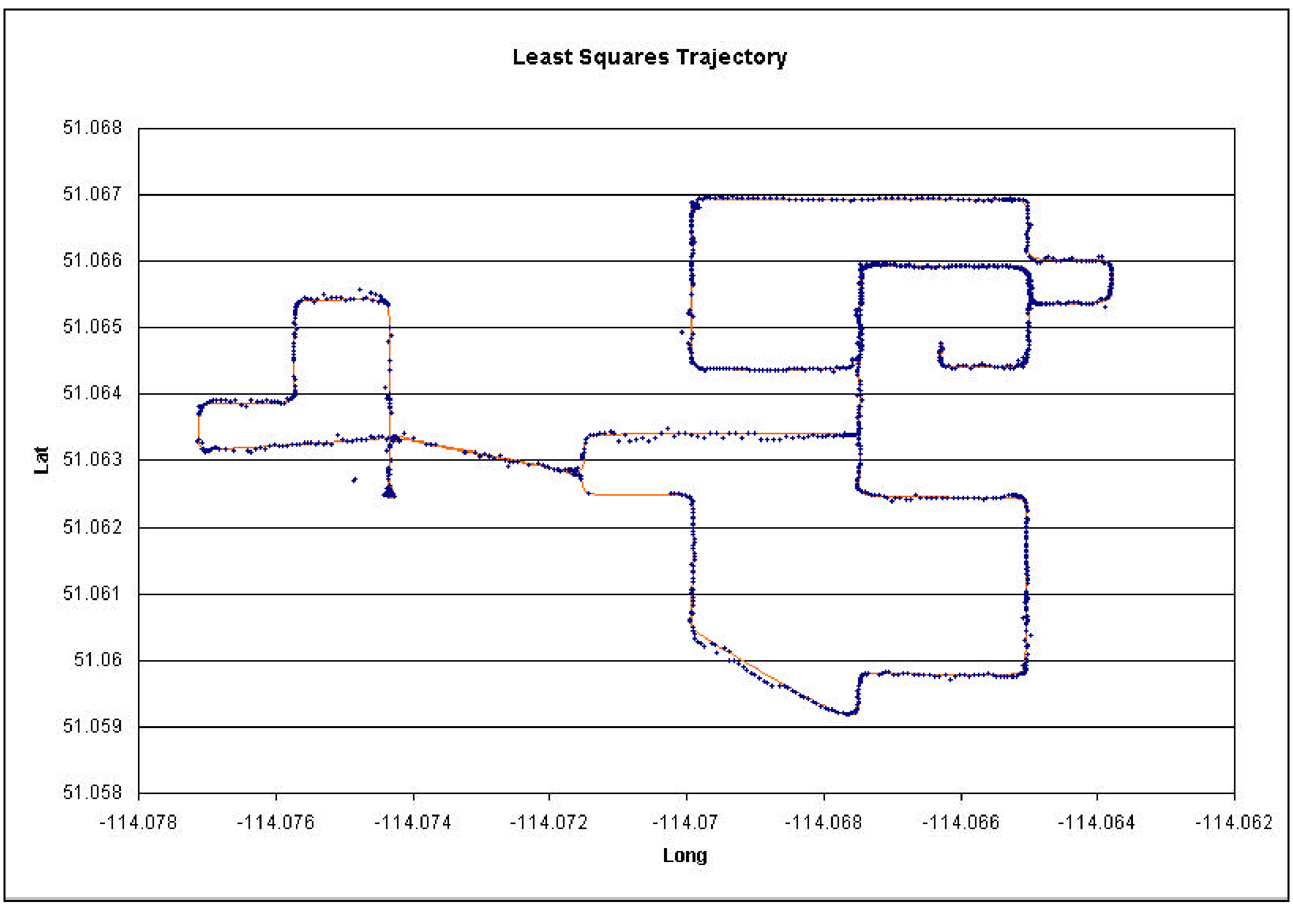
TEST RESULTS

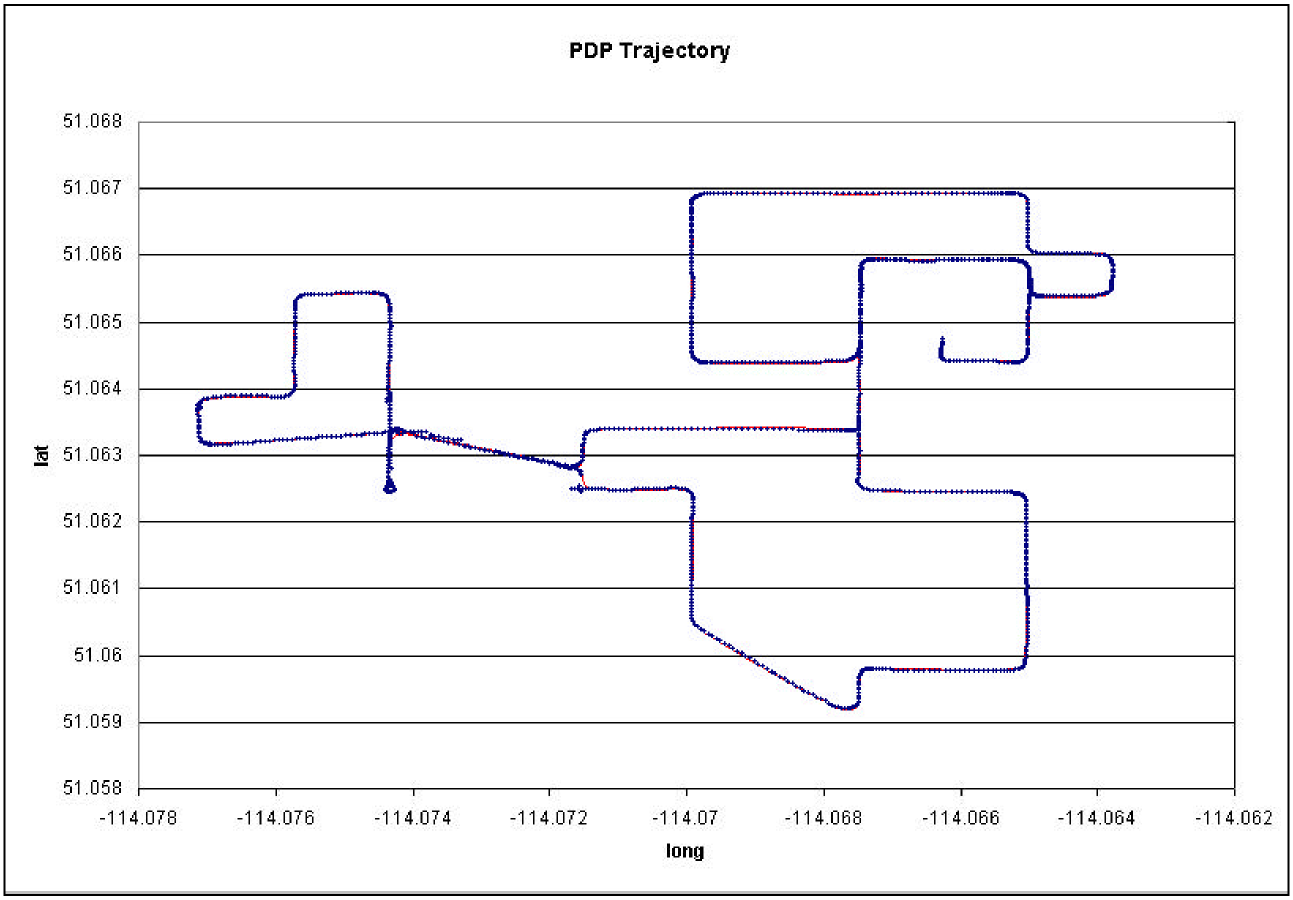
The results of the incorporation of the delta phase measurement can be seen by comparing the three sets of plots shown below. The first set shows some Crescent Heights data, and the second and third show the position improvement through downtown Calgary with its associated urban canyon geography.

Crescent Heights is an older residential neighbourhood chosen for its mature tree coverage. The coverage is seen in the following plot. Shown are the number of pseudoranges. The poor coverage later in the run corresponds to the more erratic position results seen in the west side of the trajectory plots that follow.



Now compare the least squares trajectory to the inertial control trajectory and the plot following this showing the PDP trajectory.





The PDP trajectory shows the output of the PDP Kalman filter. The result is a much smoother and accurate trajectory. The filter is also able to bridge through the portions of the test where there are fewer than four satellites in view. The maximum horizontal position error for this test has been reduced by half from over 40m to approximately 20m. The position availability percentage has increased from 87% to 100%.

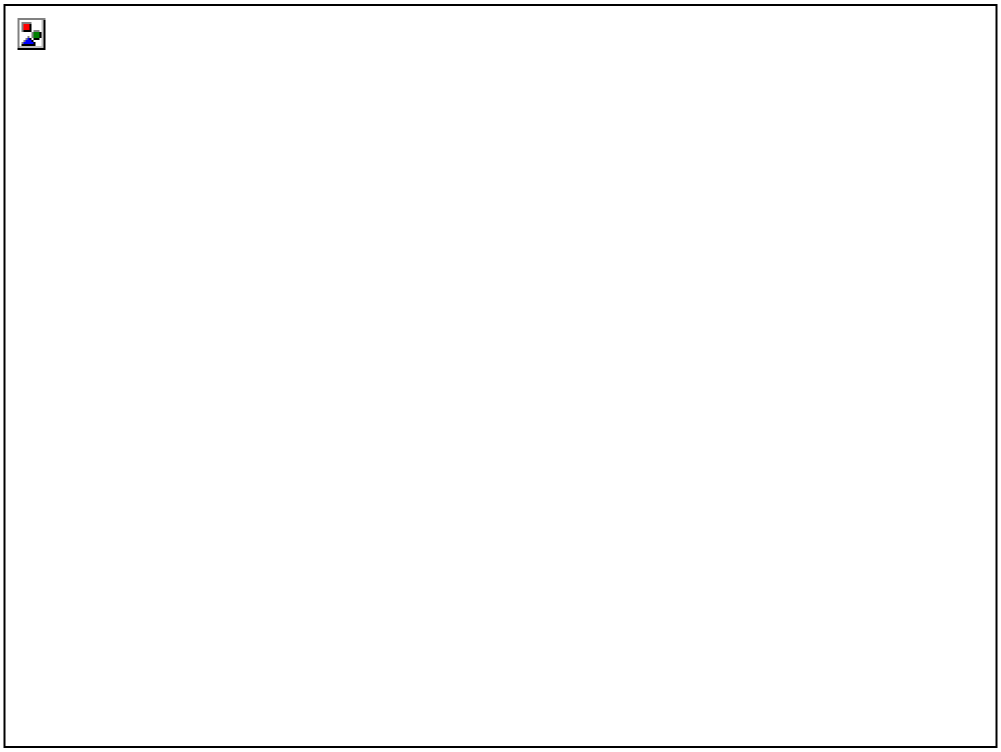
Solution Availability

	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Computed Solution Epochs	1270	1351	1459
Total Possible	1459	1459	1459
% Achieved	87	93	100

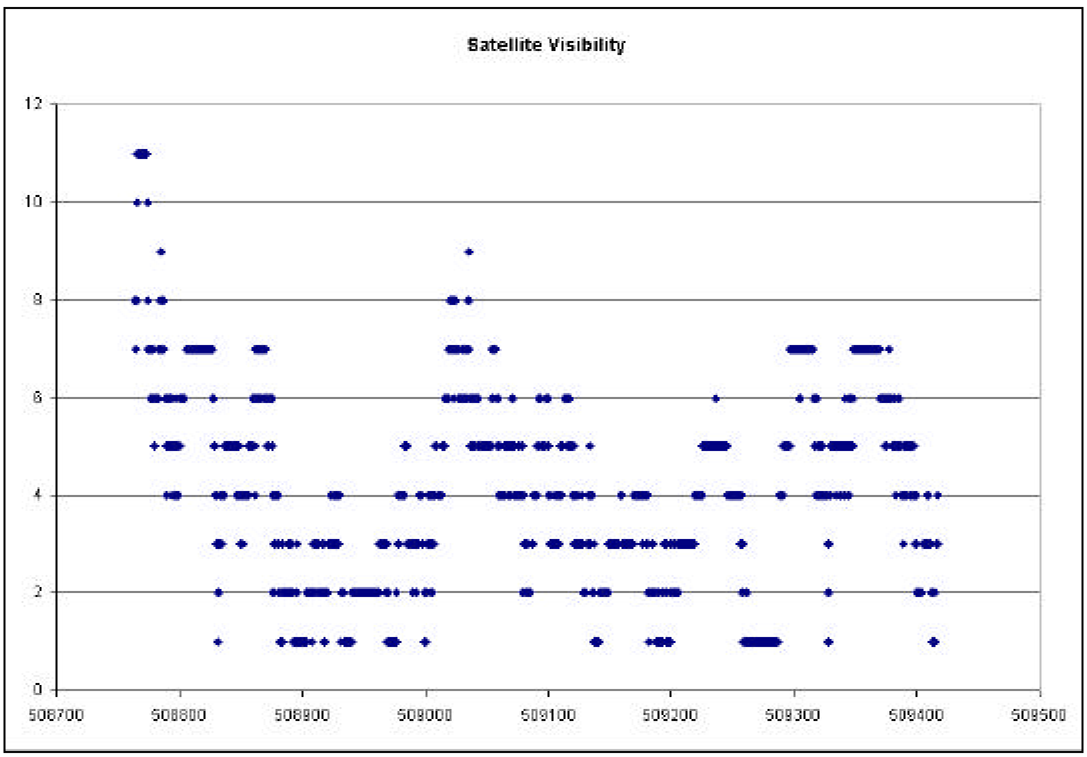
Position Accuracy

	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Latitude Error RMS	3.814	2.799	2.788
Longitude Error RMS	1.784	0.760	0.786
Height Error RMS	13.721	12.509	12.508
2D Position Error RMS	4.210	2.900	2.896

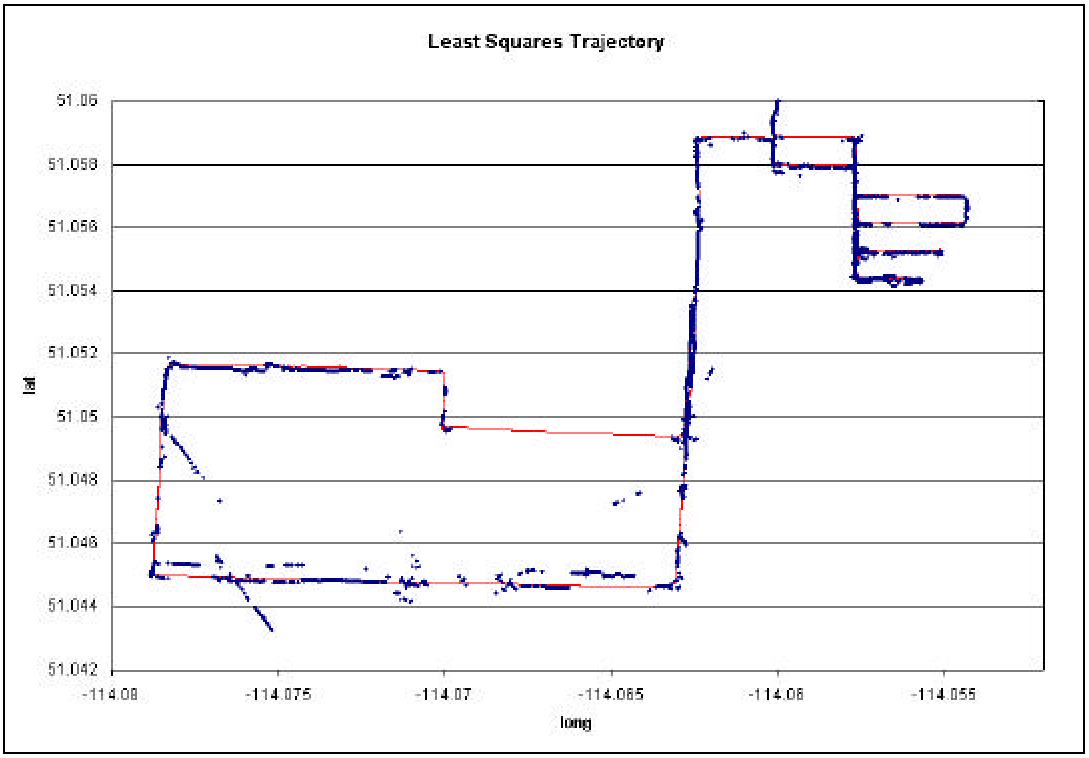
In the urban canyon setting, improvements are more evident. The following photograph and satellite availability plot show the tracking environment in the urban core. Not only is the constellation masked, but often the receiver tracks a reflected rather than the direct signal.



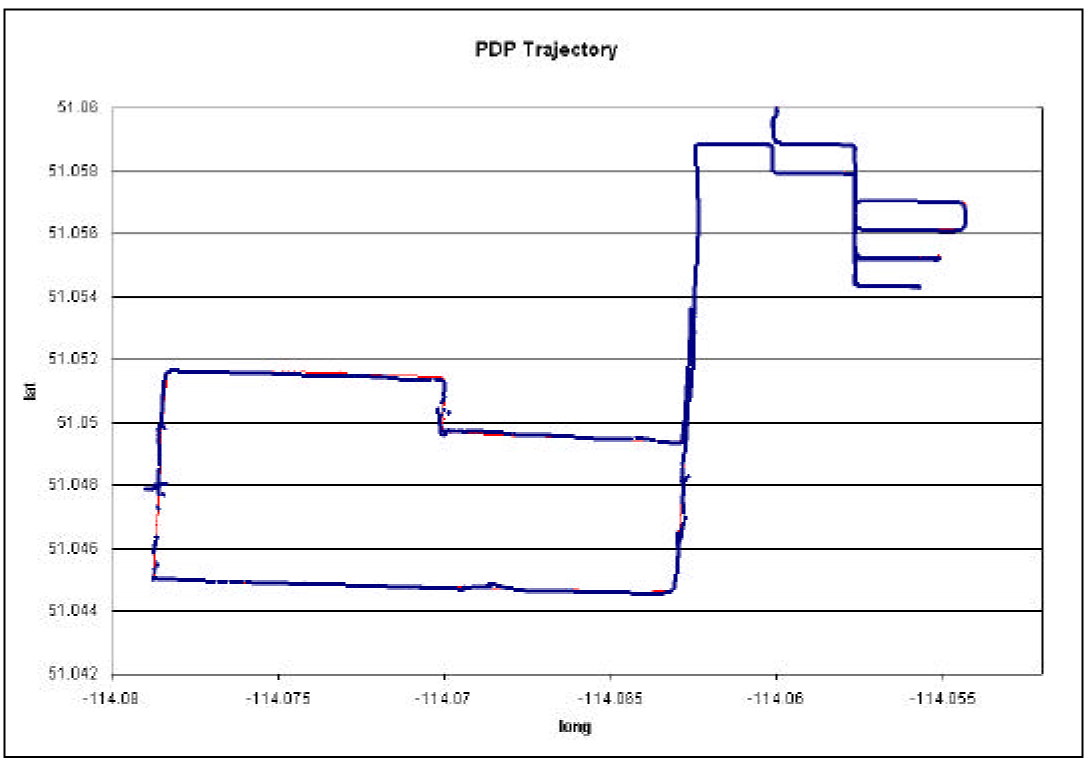
The satellite visibility plot shows that a significant proportion of the time there are fewer than four satellites available.



The next plot shows least squares derived horizontal positions in the downtown corridors.



The least squares trajectory for the first downtown data set shows very noisy data and clearly demonstrates the effect of unchecked multipath errors. Maximum horizontal position error is approaching 600m during portions of this data set.



The PDP trajectory shows the results of filtering the GPS observations. The solution availability is much improved to 99%. The maximum horizontal position error has been reduced from 600m to 95m. The position accuracy in the North/South direction is significantly higher than the component in the East/West direction. Since this test is primarily performed driving in East/West directions with high buildings on the North and South of the vehicle the satellite geometry is such that the along track direction (E/W) will be better constrained than the across track (N/S). The satellites in view will be more or less in line with the vehicle's along track direction giving relatively good control over the along track accuracy, but relatively poor control over the across track accuracy. There is one reset in the trajectory, which can be seen in the far western most portion of the southern loop. When the filter propagates without any good updates for long enough, it will reset and wait for a good least-squares solution to re-initialize. Although the availability of the least squares solution was 70% in the data shown, the availability in the true urban canyon (southern loop) was only 58%. The PDP availability during this highly shaded portion was 98%, and the horizontal RMS error was 24.7 metres.

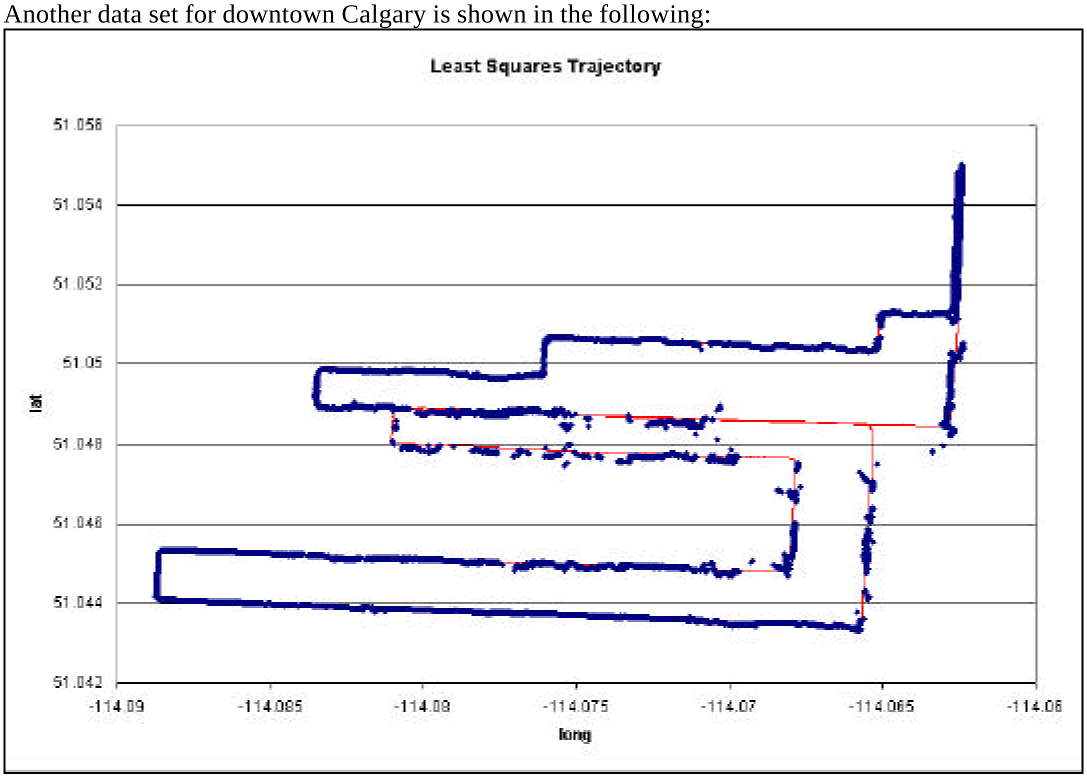
Solution Availability

	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Computed Solution Epochs	5021	6639	7103
Total Possible	7180	7180	7180
% Achieved	70	92	99

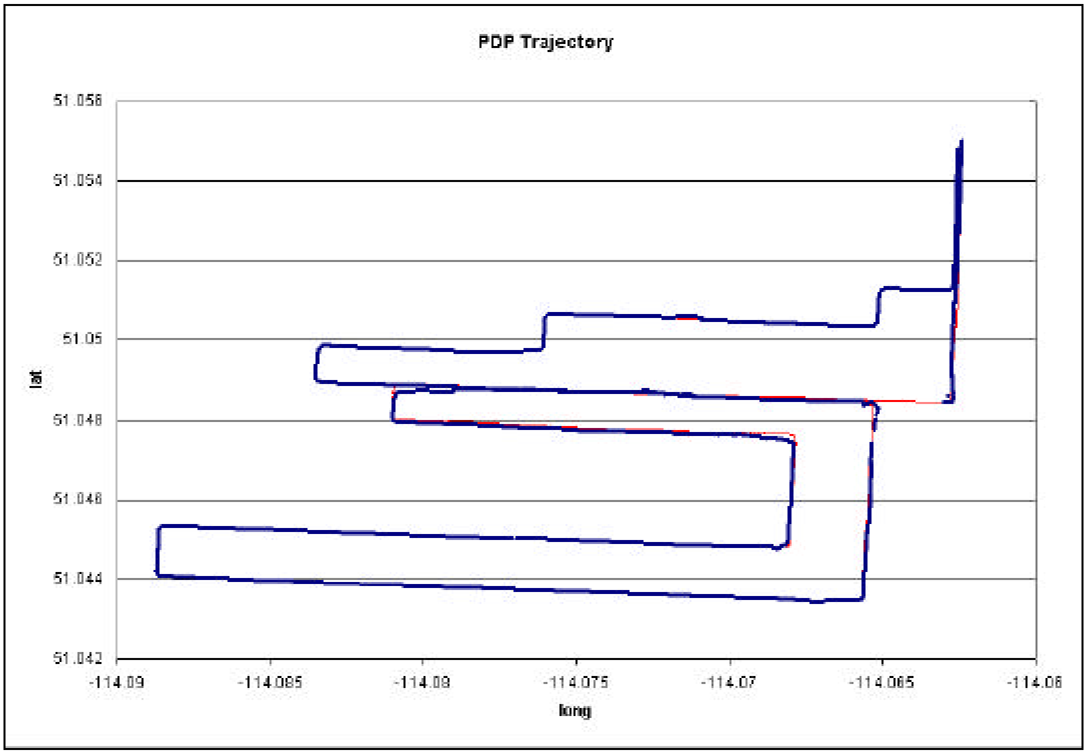
Position Accuracy

	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Latitude Error RMS	58.359	19.181	19.632
Longitude Error RMS	26.443	4.354	4.454
Height Error RMS	42.038	24.206	26.218
2D Position Error RMS	64.070	19.669	20.130

Another data set for downtown Calgary is shown in the following:



The red line shows inertial control, the blue dots show single point GPS using a least squares process with only pseudorange inputs. Compare that to the following plot of the trajectory of horizontal positions generated with a Kalman filter using pseeudorange, doppler and delta phase measurements as inputs.



The PDP trajectory plot shows the improvement in solution availability. The amount of time that a solution is not available is reduced from over 20% to only 5%. The position spikes from multipath have also been reduced. There are some small deviations from the control solution during periods when few (<4) satellites are available for extended periods of time. There is also one reset of the PDP filter in this data.

Solution Availability

	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Computed Solution Epochs	12280	14412	14749
Total Possible	15500	15500	15500
% Achieved	79	93	95

Position Accuracy

	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Latitude Error RMS	5.988	5.0172	5.4572
Longitude Error RMS	4.829	2.7322	2.7859
Height Error RMS	10.737	6.3030	6.4904
2D Position Error RMS	7.693	5.7129	6.1272

OBSERVATIONS AND CONCLUSIONS:

1) The delta phase measurement can be used with this technique in a Kalman filter to compensate for the degradation in knowledge of position from velocity error or any other time related source, to the extent that the delta carrier measurements from various satellites are known, and provided the geometry relating phase change to position change is strong enough.

2) The advantage of this technique (phase smoothing in the positioning domain) over phase smoothing in the range domain is that phase smoothed pseudo ranges require continuous tracking of a single observation for it to effectively contribute to the solution. In the implementation described above, the various satellites can lose lock and be reacquired without significant

loss in performance provided at least 4 satellites (they don't have to be the same ones) are maintained across the delta time between epochs.

3) This method has been shown to improve positioning availability in established residential neighbourhoods by over 10% and in urban canyon settings by 40%.
4) This method has improved single point horizontal accuracy from 4 metres (2DRMS) to 3 metres (2DRMS) in residential neighbourhoods. In urban canyon settings, the accuracy has improved significantly, from 64 metres (2DRMS) to 20 metres (2DRMS) in one test and from 7.6 metres (2DRMS) to 6.0 metres (2DRMS) in another.
5) The single differenced pseudoranges have significant correlation with one another due to the common errors on all observations arising from the reference satellite common to all. Doing the update as a single batch update with a fully populated pseudorange covariance matrix eliminates this issue.
6) The correlation also exists for the phase measurements. Its effect is limited by using the highest satellite as a reference, but investigations should be made to see if the performance could be improved by processing the delta phase observations in a batch process.

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FINAL NOTE

The initial development took place because Sportvision brought us a set of racing environment requirements. The happy ending to that story is that the technology has been successfully deployed by Sportvision and the results can be seen during televised NASCAR races on either FOX or NBC. A sample of the video image from FOX follows.

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