Attitude Determination in a Survey Launch Using Multi-Antenna GPS Technologies

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BIOGRAPHY

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ABSTRACT

Multi-antenna GPS receiver configurations and methods are assessed to determine roll, pitch and azimuth of a survey launch operating at cruising speeds of 10 to 15 knots. Two receiver configurations are inter-compared, namely a 4-antenna Ashtech 3DF system and a 3-GPSCardTM configuration. The survey launch antenna configurations and marine trials conducted to test these systems are described. The approach used to resolve the relative carrier phase ambiguities between the antennas is discussed and the use of antenna baseline constraints is analyzed. A least squares procedure which utilizes all the position information from the antennas for the estimation of the platform attitude parameters and their accuracy is presented. The attitude determination results of the two systems tested are inter-compared and show that the rms agreement between the two receiver configurations is better than 7 arc minutes for the pitch and yaw components and approximately 15 arc minutes for the roll component.

INTRODUCTION

The purpose of this paper is to assess the accuracy and compatibility of attitude results from an independent multiple GPSCardTM system with the 3DF system which operates all tracking channels from a single oscillator. In the marine environment multipath is relatively large and carrier phase cycle slips are generally more frequent due to the roll and pitch of the ship. Therefore, the reduction of multipath and the instantaneous or quasi-instantaneous 'on the fly' resolution of the carrier phase ambiguities are the key limitations to obtaining accurate and reliable attitude. In this paper, formulas for computing yaw, pitch and roll of a rigid body platform are analysed and a least squares algorithm which utilizes all antenna positional information for attitude estimation is given. A method for 'on the fly' ambiguity resolution which uses antenna baseline constraints is also described and tested. The data analyses show that 'on the fly' ambiguity resolution can be achieved for both the 3DF and the GPSCardTM systems within a few measurement epochs. The yaw, pitch and roll differences from the two systems are within the range of a few arc minutes to 15 arc minutes.

A'ITITUDE DETERMINATION

Attitude of a rigid body platform is determined by the orientation of the specified body frame coordinate system with respect to the reference coordinate system. In GPS attitude determination, the reference system is usually the focal-level coordinate system with the z-axis pointing upward along the ellipsoidal normal, the x-axis pointing towards the ellipsoidal east and y-axis pointing towards ellipsoidal north. The body frame platform is usually formed by choosing three GPS antennas, since three points in space define a plane. Once the body platform or the plane is defined, a body frame coordinate system can be specified within the chosen platform. For example, assuming the three antennas shown in Figure 1 form a plane, Antennas 1 to 2 can be chosen as the body frame y^b-axis. The body frame xb-axis is lying in the plane defined by Antennas 1, 2 and 3 and pointing right of the y^b-axis. The body frame z^b-axis then forms a right-handed system with the x^b and y^b axes. If the distances between the antennas are known precisely, the GPS antenna coordinates in the body frame coordinate system can be calculated immediately. This is shown for Antennas 2 and 3 in Figure 1. These body frame coordinates remain unchanged during all kinematic movements for a rigid body antenna configuration.

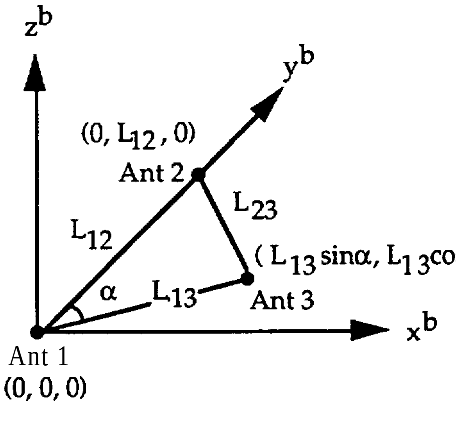

Figure 1
Body Frame Coordinate System Defined by the Three Antennas and Their Body Frame Coordinates

By using GPS carrier phase observations, the geocentric coordinates of Antennas 2 and 3 relative to Antenna 1 can be determined very precisely. These GPS-derived geocentric coordinates can then be easily transformed into the local-level system with the origin at Antenna 1 (Torge, 1980, pp. 44). Therefore, two sets of coordinates are associated with each GPS antenna. One set is the body frame system coordinates, the other set is the GPS-derived local-level coordinates. However, the local-level coordinates can be rotated to the body frame system by three consecutive right-hand rotations about the three local-level axes (e.g. Wertz, 1978; Wong, 1988). The first rotation is about local-level z-axis and the angle rotated is called the yaw of the platform. The second rotation is made about the rotated local-level x-axis and the amount rotated is the ***pitch*** of the platform. The last rotation is about the rotated y-axis and the angle rotated is the *roll* of the platform. In matrix form, the rotations read as (Wertz, 1978)

$$\begin{pmatrix} x^b \\ y^b \\ z^b \end{pmatrix} = R_2(r)R_1(p)R_3(y) \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

$$R_2(r)R_1(p)R_3(y) = R(y,p,r)=$$

$$\begin{pmatrix} c(r)c(y)-s(r)s(p)s(y) & c(r)s(y)+s(r)s(p)c(y) & -s(r)c(p) \\ -c(p)s(y) & c(p)c(y) & s(p) \\ s(r)c(y)+c(r)s(p)s(y) & s(r)s(y)-c(r)s(p)c(y) & c(r)c(p) \end{pmatrix} \tag{2}$$

where R(y, p, r) is the orthogonal rotation matrix which rotates the GPS-derived local-level coordinates $(x, y, z)^T$ of an antenna to its corresponding body frame coordinates $(x^b, yb, z^b)^T$. The functions s() and c() denote sine and cosine, respectively.

In GPS attitude determination, we know the GPS-derived local-level coordinates $(x, y, z)^T$ of all the antennas and we want to find the yaw, pitch and roll of the platform defined by the specified three GPS antennas.

Computation of Attitude Parameters

A number of formulas have been given for the computation of yaw, pitch and roll based on both the antenna's local-level coordinates and its corresponding body frame coordinates (e.g. Ashtech, 1991; van Graas and Braasch, 1991). These formulas have two disadvantages. Firstly, they require precise knowledge of all the antenna's body frame coordinates in order to compute the platform attitude parameters. This often results in the need for an initialization of the relative positions of the multiple GPS antennas before the mission. Secondly, these formulas treat the rotation matrix

R(y, p, r) as a nine independent parameter matrix and thus do not make full use of all the position information contained in multiple GPS antennas. In the case that four or more GPS antennas are in one plane, the z-component of all the antenna's body frame coordinates are zero. These formulas are then practically unusable since the matrix formed by the antenna's body frame coordinates is rank deficient which makes the computed nine parameter rotation matrix very unstable.

Two alternative methods for estimation of yaw, pitch and roll are developed each of which overcomes one of the two above problems. The first is the direct computation method which does not require knowledge of the antenna's body frame coordinates and only uses the local level coordinates of the three antennas that define the platform. The second method is a least squares estimation procedure that make full use of all the position information of multiple antennas in the local level as well as in body frame and takes into consideration the dependence of the terms in the nine parameter matrix given in Eqn. (2).

Direct Computation of Yaw, *Pitch and Roll*

Assuming that GPS Antennas 1,2 and 3 from Figure 1 form the platform, and Antennas 1 to 2 define the heading. The body frame coordinates for Antenna 2 and 3 can then be expressed as $(0, L_{12}, 0)^T$, $(L_{13}\sin(\alpha), L_{13}\cos(\alpha), 0)^T$, respectively. The corresponding GPS-derived local level coordinates for these two antennas are $(x2, y_2, z_2)^T$ and $(x3, y3, z_3)^T$. Mathematically, the body frame coordinates and their corresponding local level system coordinates for each antenna should satisfy Eqn. (1). Thus, substituting the Antenna 2 coordinates $(0, L_{12}, 0)^T$ and $(x2, y_2, z_2)^T$ into Eqn. (1) and using the orthogonality of matrix R(y, p, r), we immediately obtain the formulas for computing yaw and pitch as

$$y = -\tan^{-1}(x_2 / y_2) \tag{3}$$

$$p = \tan^{-1}(z_2 / \sqrt{x_2^2 + y_2^2}) \tag{4}$$

It can be seen from the formulas that the baseline between Antennas 1 to 2 actually determines the yaw and pitch of the platform. Once the yaw and pitch are obtained, the local-level coordinates $(x3, y3, z_3)^T$ of Antenna 3 can be first rotated around the local level z-axis by an amount y, and then rotated again around the rotated local level x-axis by an amount p. The resultant coordinates of Antenna 3 after these two rotations are denoted by (x_3'', y_3'', z_3''). A third rotation, $R_2(r)$, rotates (x_3'', y_3'', z_3'') to its body frame coordinates $(L_{13}\sin(\alpha), L_{13}\cos(\alpha), 0)^T$, namely

$$\begin{pmatrix} L_{13}\sin(\alpha) \\ L_{13}\cos(\alpha) \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(r) & 0 & -\sin(r) \\ 0 & 1 & 0 \\ \sin(r) & 0 & \cos(r) \end{pmatrix} \begin{pmatrix} x_3'' \\ y_3'' \\ z_3'' \end{pmatrix} \quad . \tag{5}$$

From the third row in Eqn. (5), roll can be computed as

r = -tan%“, $/ x_3'')$ (6)

Eqns. (3), (4) and (6) are the direct computation formulas for yaw, pitch and roll. They only use GPS-derived local-level coordinates from three GPS antennas which define the platform and thus are not dependent on *a priori* body frame coordinates. The accuracy of the computed yaw, pitch and roll can be easily derived based on error propagation laws. For instance, the computed pitch accuracy can be derived as

$$\sigma_p = \sigma_{\max} / \sqrt{x_2^2 + y_2^2 + z_2^2} = \sigma_{\max} / L_{12} \tag{7}$$

where $\sigma_{\max}$ is the maximum standard deviation of GPS-derived coordinates for Antenna 2, i.e. $\sigma_{\max} = \max(\sigma_{x2}, \sigma_{y2}, \sigma_{z2})$. It can be seen that the pitch accuracy is inversely proportional to the baseline length that defines the heading.

Least Squares *Estimation of Yaw, Pitch and Roll*

From Eqn. (2) it can be seen that the rotation matrix is solely defined by the three elements, i.e. yaw, pitch and roll. Therefore, only three elements in the orthogonal rotation matrix are independent. If the precise body frame coordinates for each antenna are known a *priori,* a least squares estimation of yaw, pitch and roll can be made based on Eqns. (1) and (2). suppose $\Theta_i^b = (x_i^b, y_i^b, z_i^b)^T$ and $\Theta_i = (x_i, y_i, z_i)^T$ are the body frame coordinates and its corresponding local level coordinates for Antenna i. Based on Eqn. (1), for all the GPS multi-antenna positions we have the following ralation

$$\begin{pmatrix} \Theta_2^b \\ \Theta_3^b \\ \vdots \\ \Theta_n^b \end{pmatrix} = R(y, p, r) \begin{pmatrix} \Theta_2 \\ \Theta_3 \\ \vdots \\ \Theta_n \end{pmatrix} \quad or \quad \Theta^b = R(y, p, r)\, \Theta \,, \tag{8}$$

with $\Theta^b = (\Theta_2^b, \Theta_3^b, \cdots, \Theta_n^b)^T, \Theta = (\Theta_2, \Theta_3, \cdots, \Theta_n)^T$. In Eqn. (8), Θ^b is known a *priori* with a covariance matrix $Var(\Theta^b)$. Θ is treated as our observations which are derived from GPS carrier phase measurements with the covariance matrix $Var(\Theta)$. The unknown parameters to be resolved are (y, p, r). Such a model is a standard *implicit* least squares adjustment model and the solution is described in Krakiwsky (1987).

From a statistical aspect, the least squares estimation of yaw, pitch and roll gives the best estimates based on all the position information contained in a multiple GPS antenna array. Another advantage of least squares estimation over the direct computation is that the least squares solution is less affected by multipath on a single antenna since the solution is made by the best fit over all antenna positions.

ON-THE-ELY AMBIGUITY RESOLUTION

The accuracy of the estimated attitude parameters mainly depends on the relative antenna positions in the local level system. In order to achieve high accuracy positioning it is necessary to resolve the correct integer carrier phase ambiguities 'on the fly'. In a marine survey launch, the ship can never be static even if the ship is anchored in the harbour. Therefore, on-the-fly ambiguity initialization is needed at the beginning of the mission as well as on occasions **when** multiple cycle slips occur. For a real-time system, it is also required that the ambiguities be resolved as efficiently as possible.

The technique used herein for on-the-fly ambiguity resolution is a variation of a method proposed by Hatch (1991). The combination of double difference integer ambiguities from four primary satellites are tested to determine which combination gives the best fit to the data sequence in a least squares sense. In addition to the test of variance factor calculated from the sum of squared residuals, the measured baseline lengths between antennas introduce additional constraints to isolate the correct integer ambiguity set. It is found that the baseline length known to within a few centimetres can significantly speed up the ambiguity search process and increase the reliability of the process. The known baseline length is also useful for cycle slip detection during the system's normal operation since a cycle slip in the carrier phase will cause the computed baseline length to deviate from its true value. The third test included in this procedure for on-the-fly ambiguity resolution is the ratio test which eliminates the possible effect of the a priori carrier phase variance $(\sigma^2 \nabla\Delta\phi)$ on the results. The ratio of two smallest variance factors is computed. If it is greater than a preset value, the potential ambiguity set with the smaller variance is selected as the correct ambiguity set. For more information about on-the-fly ambiguity resolution method used herein, refer to Lachapelle et al (1991,1993) and Cannon et al 0992).

MARINE SURVEY LAUNCHTEST

A marine survey launch test was jointly conducted by the Canadian Hydrographic Service (CHS) and The University of Calgary on September 3, 1992 off the coast of Sidney, British Columbia. The 4-antenna 3DF system of The University of Calgary (e.g., Schwarz et al 1992) and a 3-GPSCard™ system were set up on a 12 m survey launch. All the antennas were mounted on two wooden beams placed across the width of the bow and stem of the boat. Baseline lengths between the antennas were measured with an accuracy of 1 cm. The antenna configurations are shown in Figure 2.

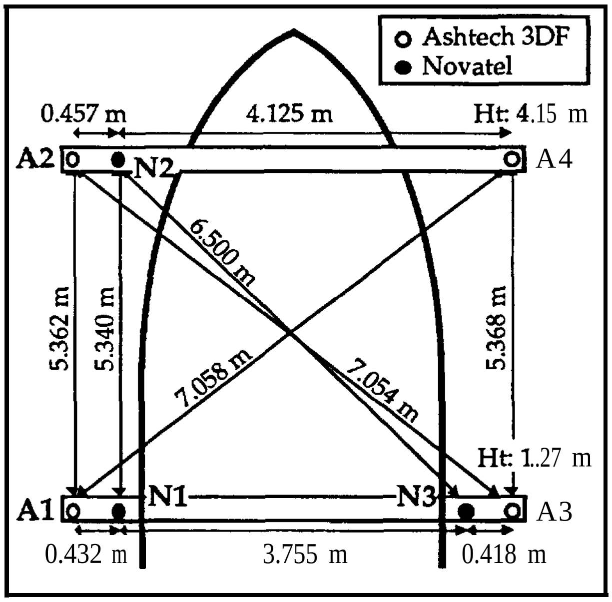

Figure 2
Survey Launch GPS Antenna Configuration

During the test, approximately 15 minutes of data was collected while the boat was tied up to the wharf. The boat then traveled at cruising speeds of 10 to 15 knots for approximately three hours during which up to six satellites were tracked. The 3DF raw data was internally logged while the three GPSCard™ data were housed in two Grid laptop computers (two receivers were in one Grid) and data was recorded on the computer's hardrive.

GPSCardTM Antenna's 1 and 3 had chokering ground planes while Antenna 2 had none. One hour of data in the middle of the test was selected for post-processing, where six satellites above 15 degrees were tracked by all antennas. The survey launch trajectory for the selected data is shown in Figure 3.

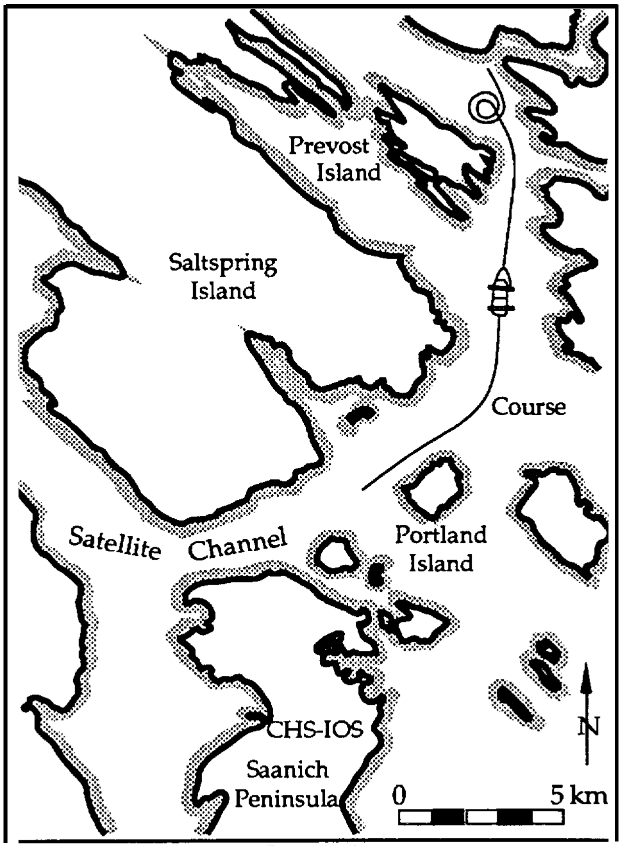

Figure 3
Survey Launch Trajectory

For attitude computation, it is assumed herein that the platform plane is defined by Antenna's 1,2 and 3 of the 3DF system. The direction from Antennas 1 to 2 defines the ship's heading, i.e. y^b-axis in the antenna's body frame coordinate system. In this case, the yaw and pitch of the ship are determined by the baseline from Antennas 1 to 2. It can be seen from Figure 2 that the two wooden beams were not placed in the same horizontal plane. This results in a pitch bias angle of 32.4873 degrees. In the following computation, this bias angle was subtracted from the estimated pitch of the platform.

The above test also include Ashtech P-XII receivers operating in DGPS mode. This test was conducted to assess the capabilities of P code and high performance C/A code for ambiguity resolution on the fly in the marine environment. The results are reported by Lachapelle et al (1993).

DATA PROCESSING AND RESULTS

In order to process and compare the attitude results of the 3DF and GPSCardTM systems, a program has been developed to process both the 3DF and the multiple GPSCardTM raw data. Carrier phase double difference observables are used to derive the precise relative antenna positions of the attitude system. Although the 3DF system operates all its tracking channels from a single oscillator, residual receiver clock errors still existed making single difference processing unusable.

This program accepts the measured baseline lengths between multiple antennas and resolves the double difference (DD) carrier phase ambiguities on the fly so no static initialization is required. Once the DD ambiguities are resolved for all the antenna baselines, the yaw, pitch and roll are estimated at each measurement epoch. If the antenna's body frame system coordinates are known a priori, the least squares estimates of yaw, pitch and roll are then computed by the formulas previously given. Carrier phase cycle slips are first checked using a prediction technique which utilizes the Doppler frequency observable. A more rigorous test for cycle slips is made by checking the difference between the measured antenna baseline length and the GPS computed baseline length. If the difference is larger than a preset value, the ambiguities related to that baseline are considered invalid and ambiguity resolution is re-initialized. Clearly the higher the accuracy of the measured baselines between multiple antennas, the better the ability to detect small cycle slips in the carrier phase measurements. A tolerance of a few cm was used in this case.

Attitude Determination Results

As previously discussed, about one hour of data from the middle of the test where the GDOP ranged between 2 - 3 was selected for post-analysis. The ambiguity search volume for the 3DF was determined by the baseline lengths between the antennas since 3DF is a standard C/A code receiver. This leads to a search volume for some antenna pairs of over 10 m on a side, i.e. the unknown antenna location is within ± 5 m of. the known antenna location for a 5 m separation. Since the GPSCard™ system is a high performance C/A code receiver which has a 10-cm code accuracy, the ambiguity search interval was set at ±15 cycles around the

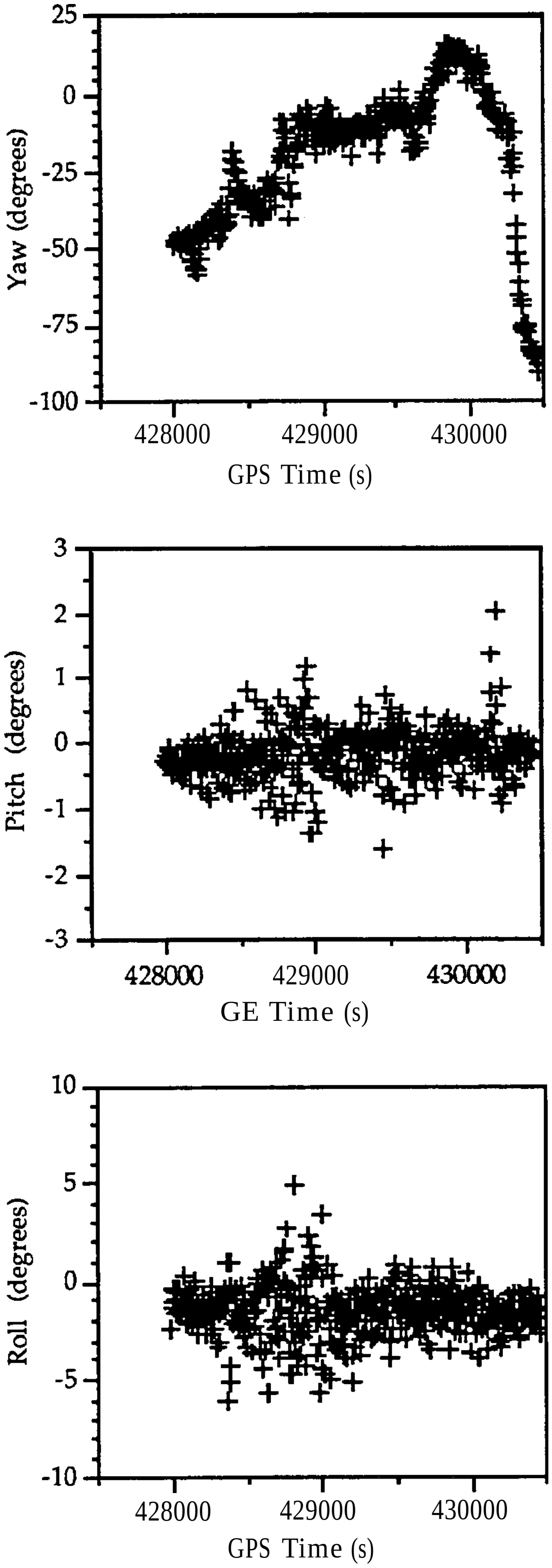

Figure 4
Yaw, Pitch and Roll of the Survey Launch Derived from the 3DF System

carrier phase ambiguities derived from the code solution, i.e. f 15 cycles is approximately ± 2.8 m. The ambiguities for the 3DF system were resolved in three kinematic epochs at initialization, while the ambiguities for GPSCard™ system were resolved using 11 epochs of kinematic observations. The longer time required for the GPSCard™ system was due to a number of factors, namely that it was a three antenna system and the baselines measured between GPSCard™ antennas were less accurately measured than between 3DF antennas.

Figure 4 shows the yaw, pitch and roll of the ship obtained from the 3DF system. The pitch bias angle caused by the two wooden beams at different heights was removed from the results.

In order to compare the attitude of the 3DF system with the GPSCard™ system, the ideal situation would be that both platforms were perfectly parallel in space so the two systems could be compared directly. However, the antennas from each system were not placed exactly on one line on the wooden beams, nor was the heading baseline of the 3DF system parallel to that of the GPSCard™ system. Therefore, orientation differences existing in yaw, pitch and roll between the two platforms had to be determined before the results could be compared. This was achieved by selecting the 3DF antenna platform as a reference system. The GPS-derived local level coordinates of GPSCard™ antennas at each epoch are then rotated to the body frame coordinate system defined by the 3DF antennas using the yaw, pitch and roll values from the 3DF system. The coordinates of the GPSCard™ antennas in the 3DF body frame coordinate system then determines the orientation differences between the two platforms. Since the two platforms are considered as rigid body platforms, a least squares estimation of the orientation differences can be made if more than one epoch of data are available. based on approximately 40 minutes data (1 Hz data rate), the estimated orientation differences between the two platforms in yaw, pitch and roll are -18.424 minutes, -23.458 minutes and 40.042 minutes, respectively. The estimated yaw difference is very close to the value -16.028 minutes which was computed by the measured distances between the 3DF and GPSCard™ antennas, as shown in Figure 2. Once the orientation biases between the two platforms are determined, the attitude results from the two systems can be compared. Figure 5 shows the differences in the attitude results obtained from the two systems.

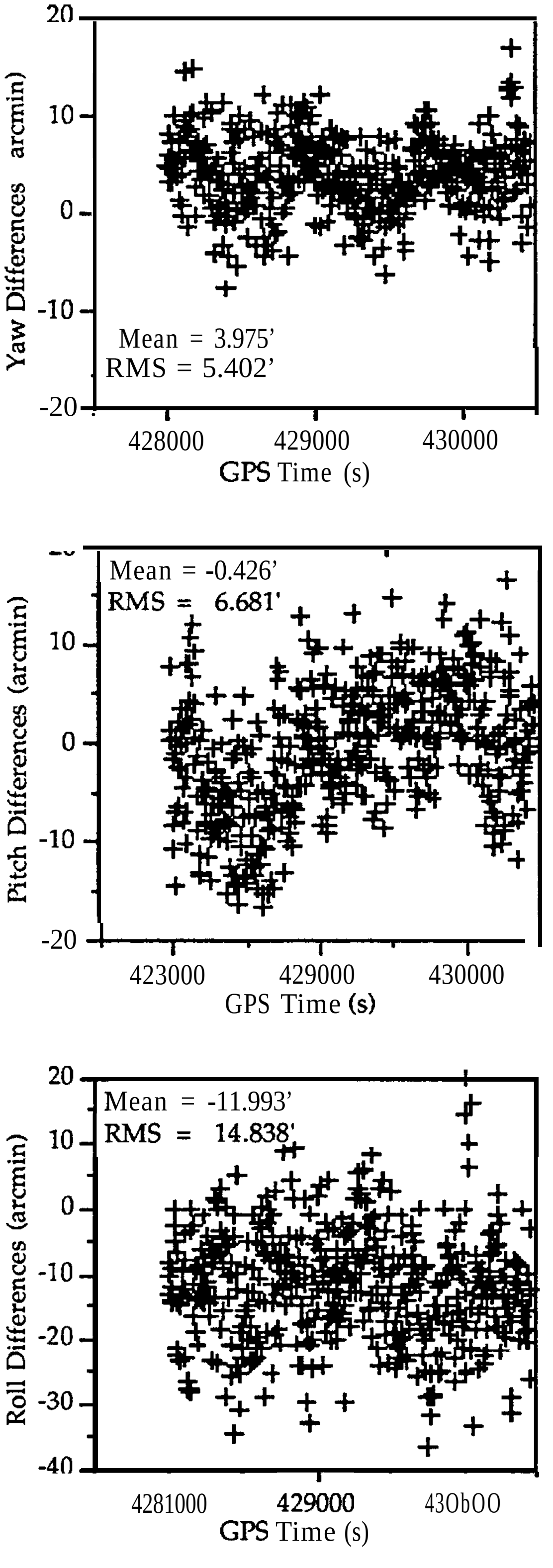

Figure 5
Attitude Differences between the 3DF and GPSCard™ System

Shown in Table 1 are the summarized statistics for the attitude determination results. The agreement of the estimated attitude parameters from the two systems are at the range of 5 arc minutes to some 15 arc minutes (rms). This level of agreement approaches the limit of the achievable accuracy of differential GPS positioning. For example, a 5 mm relative positioning accuracy will result in an accuracy of 3.4 arc minutes over a 5 metre baseline for pitch estimation. The results shown in Table 1 also indicate that the attitude determination accuracy from the cost-effective integrated multiple GPSCard™ system is comparable to the dedicated 3DF system.

Table 1
Attitude Differences Between the 3DF and the GPSCard™ System

Attitude	LS Solution		Direct Computation	
	Mean (arcmin)	RMS (arcmin)	Mean (arcrnin)	RMS (arcmin)
Yaw	3.975	5.402	5.349	6.587
Pitch	-0.426	6.681	2.194	8.089
Roll	-11.993	14.838	-12.626	16.215

From Table 1 it can be seen that the agreement of the least squares estimates of the attitude parameters for the two systems are slightly better than those from the direct computation. This is because the additional information of the antenna's body frame coordinates are used in LS estimation. In this project, the antenna body frame coordinates used in LS estimation for both systems are obtained by averaging the GPS relative positioning results of the multiple antennas over 45 minutes of data with integer ambiguities resolved.

Shown in Figure 6 are the SVs 23-21 carrier phase double difference residuals with fixed integer ambiguities for the 3DF and GPSCard™ systems. Note that the residuals for Antennas l-3 have been offset by -1 cm for clarity. For the GPSCard™ antennas, only Antenna 2 had no chokering ground plane. Slightly higher multipath influences on the residuals are evident for the baseline between GPSCard™ Antennas 1 to 2. The different multipath signatures from the two systems without chokerings likely arises from the use of different types of antennas.

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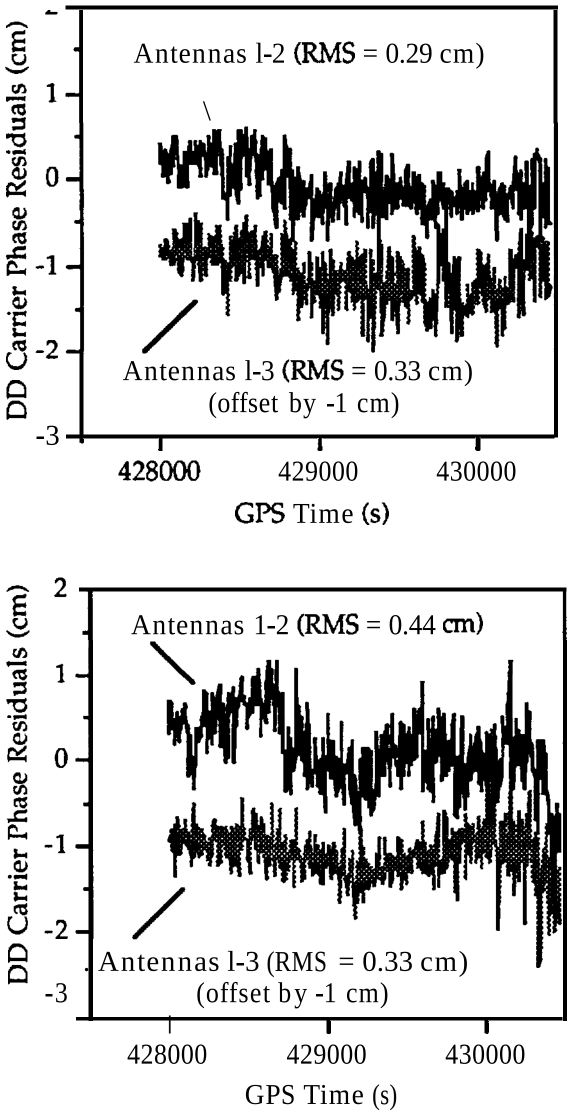

Figure 6
Double Difference Carrier Phase Residuals of SVs 23- 21 for the 3DF (above) and the GPSCard™ (below) Systems

The observation accuracy from the 3DF system is slightly better than the accuracy of the GPSCard™ observations without chokering ground planes. With chokerings, the carrier phase accuracy from both systems is almost at the same level. To achieve the ultimate attitude determination accuracy, multipath influences on the carrier phase observations should be reduced as much as possible.

CONCLUSIONS

GPS attitude determination in the marine environment has been investigated using two equipment configurations, namely a dedicated 4-antenna 3DF system and a 3-GPSCard™ system. The results show that the agreement of the attitude parameters estimated from these two systems is within 5 to 15 arc minutes level (rms) in a relatively strong multipath environment. The accuracy of either system would then be $\sqrt{2}$ smaller, i.e.., 3.5 - 10 arc minutes. The attitude determination accuracy from the multiple GPSCard™ system used herein appears to be comparable with that of a dedicated GPS attitude determination unit that operate all tracking channels from a single oscillator.

The derived theoretical formulas for yaw, pitch and roll estimation are simple, robust and reliable. The least squares estimation procedure is optimal for attitude determination in the sense that it makes full use of all the position infornation contained in the multiple antenna system. However, the body frame coordinates of the antennas should be known *a* priori for the least squares estimation procedure to be used.

On-the-fly ambiguity resolution is one of the most important aspects to ensure that the GPS attitude determination system is working properly. Test results have demonstrated that the ambiguity resolution method described in this paper works effectively The use of antenna baseline constraints significantly increased the speed and the reliability of ambiguity resolution. They also provide a reliable and rigorous check for cycle slips which can severely degrade the achievable attitude accuracy.

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