Analysis of the Multipath Meter Performance in the Presence of Anomalous Satellite Signals

by

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BIOGRAPHY

Andy Jakab received his B.Sc. in 1997 from the Geomatics Engineering department at the University of Calgary and is currently pursuing a masters degree. After receiving his undergraduate degree, he joined NovAtel and has worked on receiver testing and end user software. Andy is currently a GPS Systems Engineer for the Aviation group working on receiver certification and development activities including evil waveform research.

Bryan Townsend received his Bachelors of Science in 1989 and Masters of Science in 1993 from the Department of Geomatics Engineering at the University of Calgary. Since then, he has worked in several areas of GPS including GPS surveying, and GPS receiver design. Currently he is working with wide area reference systems such as WAAS and EGNOS.

ABSTRACT

Anomalous GPS satellite signals can have a very unfavorable impact on GPS receiver integrity. In either a wide area augmentation system (WAAS), or local area augmentation system (LAAS) any differential corrections sent to other users must accurately reflect the current satellite signal integrity. Users of these systems must be warned of potentially misleading information, including both satellite failures and / or multipath, by a specific scheme designed for satellite failures detection (SFD) in the event of such an occurrence. Excessive multipath at the reference station antenna may also have an adverse impact on users of WAAS and LAAS.

The Multipath Estimating Delay-Lock-Loop (MEDLL) is a method for mitigating the effects due to multipath within the receiver tracking loops. The traditional implementation of the MEDLL has been with a narrower RF bandwidth receiver that is adequate for multipath estimation but limits the detectability of anomalous satellite signals. By implementing the MEDLL algorithms on a wider bandwidth receiver, the ability to estimate multipath signals should improve as well as the ability to detect anomalous satellite signals. A theoretical multipath error envelope for this new implementation is presented.

The Multipath Meter (MPM) has been described in previous papers and been shown to be a good tool for real time signal quality monitoring (SQM) and multipath detection. In order to measure multipath the MPM outputs the delay, relative amplitude, and phase of the multipath signal along with the residual values for each of correlator used in the estimation process as well as its raw and multipath corrected correlator measurements. The results show a theoretical analysis of how the multipath meter outputs are affected by various GPS satellite signal failures and the ability of the MPM to detect these satellite failures.

INTRODUCTION

The Multipath Estimating Delay-Lock-Loop (MEDLL) is a method for mitigating the effects of GPS signal multipath within the receiver tracking loops. The MEDLL does this by separating the signal into its lineof-sight and multipath components. The line-of-sight component is then used for computing the tracking error. The MEDLL currently estimates only one multipath signal. The MEDLL receiver performance, in mitigating the effects of multipath on C/A code pseudorange and carrier phase measurements, has been demonstrated in previous papers [2, 3, and 7].

The MEDLL receiver was modified to output the multipath parameters. The name 'Multipath Meter' comes from the fact that it 'measures multipath'. The parameters outputs include the delay, relative amplitude, and phase of the multipath signal, along with the residual

values for each correlator. The performance of the Multipath Meter has been demonstrated in previous papers [8 and 9]. Also available for output are raw and multipath corrected correlator values that can be used for satellite failure detection. The multipath parameters are estimated by the MEDLL and the residuals indicate the quality of the estimation process. The correlator values are what MEDLL uses to estimate the multipath signal and the corrected correlator values are those same values with the estimate of the multipath signal removed.

GPS MULTIPATH

The term multipath is derived from the fact that a signal transmitted from a GPS satellite can follow a 'multiple' number of propagation 'paths' to the receiving antenna. This is possible because the signal can be reflected back to the antenna off surrounding objects, including the earth's surface. Figure 1 illustrates this phenomenon for one reflected signal.



Figure 1: Direct Path and Multipath (Reflected Path) Signals

Some important characteristics of multipath are as follows [4]:

- i) The multipath signal will always arrive after the direct path signal because it must travel a longer distance over the propagation path.
- ii) The multipath signal will normally be weaker than the direct path signal since some signal power will be lost from the reflection.
- iii) If the delay of the multipath is less than two PRN code chip lengths, the internally generated receiver signal will partially correlate with it. Generally, if the delay is greater than 1 chip the correlation power will be negligible.

MEDLL

In the presence of multipath propagation, the received signal at the input of a direct-sequence spread-spectrum receiver can be written as:

$$r(t) = \sum_{m=0}^{M-1} a_m p(t - \tau_m) \cos(\omega t + \theta_m) + n(t)$$
(1)

Where,

Μ	=	number of signals
t	=	time
p(t)	=	the spread-spectrum code
n(t)	=	white Gaussian noise
a_m	=	component signal amplitude
τ_m	=	component signal delay
θ_m	=	component signal phase

For a positioning system like GPS, the parameters of interest are the direct path signal delay and phase. In order to estimate these parameters, the direct path correlation function needs to be determined. The MEDLL approach used here involves the decomposition of the correlation function into its direct path and reflected path components.

The MEDLL estimates the amplitude, delay, and phase of each multipath component using maximum likelihood criteria. Each estimated multipath correlation function component is in turn subtracted from the measured correlation function. The result is an estimate of the direct path correlation function. A standard early-late DLL is applied to the direct path component of the correlation function giving a 'multipath free' estimate of the code loop tracking error.

THE MULTIPATH METER CONCEPT

The multipath meter concept involves taking the signal parameters output from the MEDLL algorithms and using them for quality monitoring of the GPS signal. These parameters are the delay, relative amplitude, and phase of the multipath signal along with the residual values for each correlator. Also available are multipath corrected and uncorrected correlator measurement values.

The multipath meter output is used for real-time signal quality monitoring. It can also be used for reference station site surveys to determine if a location is suitable for a GPS reference station. Of the signal parameters output, the amplitude of the multipath is of most interest because it will indicate the presences of multipath even if it is not causing any pseudorange error. This would be the case if the relative phase of the multipath is 90 or 270 degrees.

THEORETICAL PERFORMANCE

Figure 2 shows a plot of the pseudorange error due to multipath for MEDLL (implemented on both an 8MHz and 16MHz bandwidth receiver) along with the Narrow Correlator tracking (16 MHz bandwidth receiver). The lightest (yellow) colored line is the 16 MHz MEDLL, the medium (pink) colored line in the 8 MHz MEDLL, and the dark (blue) colored line is the Narrow Correlator. The multipath signal has an amplitude (a_m) of 0.5 relative to the direct path signal. The delay is varied from 0 to 1.1 chips. The plot appears almost solid because the multipath varies in phase as the delay is varied. One carrier phase cycle is 1/1540 chips and as the delay varies over one cycle the phase of the multipath signal relative to the direct path signal varies over 360 degrees. The multipath error is at its maximum positive value at 0 degrees phase and at its maximum negative value at 180 degrees phase. The error is 0 at 90 and 270 degrees.



Figure 2: Theoretical pseudorange multipath error for MEDLL and Narrow Correlator ($a_m = 0.5$)

The variations in the bandwidth of the MEDLL receiver were simulated in order to observe the current NovAtel WAAS receiver implementation (8 MHz pre-correlation bandwidth, OEM2 hardware) as well as the next generation receiver design in which MEDLL may be implemented (16 to 24 MHz pre-correlation bandwidth, OEM4 hardware). The increased bandwidth of the 16 MHz MEDLL implementation also allows for a more robust detector or anomalous satellite signals. Since there is less rounding of the correlation function due to filter characteristics, the ability of the receiver to detect anomalous signals is improved. This wider filter also allows for the correlators to be placed closer to the peak of the correlation function, further reducing the effects of multipath. Preliminary results of implementing the MEDLL algorithms on a 16 MHz bandwidth receiver have been done (on NovAtel's OEM3 hardware). However, these results are beyond the scope of this theoretically focused paper.

We can see that for the 16 MHz implementation of the MEDLL receiver, the maximum error due to multipath is just below 3m and occurs at a delay of 0.05 chips (15 meters). Beyond this delay, the multipath errors decrease substantially until they are negligible beyond delays of 0.125 chips (37.5 meters). This represents a reduction in the effects of multipath by a factor of 2 over the current 8 MHz MEDLL implementation.

For the same multipath scenario as in Figure 2, the corresponding theoretically measured multipath power is shown in Figure 3 for both MEDLL implementations.



Figure 3: Theoretical D/U values for 0.5 amplitude signal

The multipath power is plotted in D/U (desired signal power over undesired signal power) in decibels (dB). The D/U is calculated using the equation,

$$D/U = 20*\log(a_{direct} / a_{multipath})$$
(2)

The plot shows that the D/U converges to 6 dB as the delay of the multipath is increased. 6 dB corresponds to a relative multipath amplitude of 0.5. For delays less than 0.2 the D/U for the 8 MHz MEDLL becomes less accurate. Whereas for the 16 MHz MEDLL implementation, the D/U only becomes less accurate with

multipath delay less than 0.125 chips. This shows that the accuracy of the MEDLL is limited by the BW of the GPS receiver RF deck and the GPS signal itself. The 16 MHz MEDLL converges much quicker to the correct D/U estimate than the 8 MHz MEDLL.

NovAtel has also developed specific software (Multipath Assessment Tool, MAT [9]) for use with the Multipath Meter to examine its outputs.

ANOMALOUS SIGNALS

GPS satellite failure modes in which WAAS and LAAS ground station receivers must protect other users from have been defined [11] through research by the RTCA (Requirements, Technology and Concepts for Aviation).

There are three main types of signal failures, denoted by threat models A, B, and C. Threat model A involves the digital failure of the satellite signal. These failures produce a lead or lag of the falling or rising edge of the C/A code chip transitions. An example of this is shown in Figure 4.



Figure 4: Threat Model A chip effects

Threat model A can produce these lead/lag effects within the confines of $-0.12 \le \Delta \le 0.12$ [11]. Where Δ is the lead/lag offset of the chip transition from its nominal value.

Threat model A effects can generate flat correlation peaks that may result in multiple tracking regions depending on the receiver design.

Threat model B is characterized by a second order step response as seen in the equation below:

With $4 \le f_d \le 17$ and $0.8 \le \sigma \le 8.8$ [11].

$$e(t) = 1 - e^{(-\sigma)} \left[\cos(\omega_d t) + \frac{\sigma}{\omega_d} \sin(\omega_d t) \right]$$
(3)

Threat model B produces "ringing" of the chip transitions so that they are not as expected. This ringing can result in distorted correlation peaks. This ringing is effect is shown in Figure 5.



Figure 5: Threat Model B chip effects

Threat model C can produce some of the most "evil" waveforms, because it is a combination of both threat models A and B together. The treat space is slightly smaller then the combined areas of threat models A and B, with the variables in the range $-0.12 \le \Delta \le 0.12$, $7.3 \le f_d \le 13$ and $0.8 \le \sigma \le 8.8$ [11].

Threat model C can produce flat peaks, distorted peaks, and multiple peaks in the correlation domain. Much effort has been invested in determining methods of detecting these anomalous signals. The general consensus has been that detection will be derived from multiple correlation function measurements so that the integrity of the correlation function can be assured. However, when observing the correlation function in this manner, the measurements are easily corrupted by multipath [12].

MDE AND MDR VALUES

A Minimum Detectable Error (MDE) refers to our ability to detect when a single correlation value is in error. This error can be either above or below what the expected value of the correlation should be given the location of the correlator from the punctual code measurement. We have both a high and low threshold of detection in order to determine when there is an error in the measured signal. It is important to verify this value at numerous correlator locations in order to improve our ability to detect anomalous signals. In addition, different correlator locations are effected differently by different evil waveforms.

A Minimum Detectable Ratio (MDR) also refers to out ability to detect signal failures. However, with this value we are looking at the variation across the peak of the correlation function. By design, our code-tracking loop is trying to force one of these ratios of early and late correlator values to zero. What we are using as a detection mechanism here is the ratio of other non-tracking correlator pairs, also across the peak of the correlation function. These values are subtracted from the error observed from the tracking pair as a means of removing any inherent tracking offset and focusing on the actual signal anomaly, which may or may not be present.

ANOMALOUS SIGNALS WITH MULTIPATH

The next step in our analysis is to generate these satellite signal failures for use in simulation with the Multipath Meter. All further simulations involve comparisons using the 16 MHz MEDLL implementation and its multipath corrected and uncorrected correlator values. All multipath testing is for a multipath signal with amplitude $a_m = 0.5$.

After generating the evil waveforms for all points in the threat space, the distorted correlation functions were used to create multipath signals. These evil multipath signals were then used as input into the MEDLL algorithms. Unbeknownst to the MEDLL, it still attempts to remove multipath effects using a nominal reference correlation function (i.e. a reference without any evil waveforms) when there is evil present.

Three specific points were examined in the threat space and are presented here, with one point being chosen from each of the threat models. The threat model parameters were picked so that they would introduce some of the most distorted waveforms (maximum Δ , minimum f_d and minimum σ). For evaluation of the Multipath Meter, the D/U estimate, correlator residuals, and multipath corrected correlator values are examined for their abilities to detect the evil waveforms.

The MEDLL correlator residuals are calculated by taking the measured correlator value and subtracting off the computed direct path and multipath correlator values. The direct path and multipath correlator values are calculated using a pre-determined reference correlation function. The correlator residual is represented by,

$$C_{\rm res} = C_{\rm meas} - \sum_{m=0}^{M-1} a_m C_{\rm ref} (x - \tau_m) \cos(\theta_m)$$
(4)

Where,

$C_{\rm res}$	=	correlator residual
$C_{\rm meas}$	=	measured correlator value
$C_{\rm ref}$	=	reference function correlator value
Μ	=	number of signals
x	=	correlator position
a_m	=	component signal amplitude
τ_m	=	component signal delay
θ_m	=	component signal phase

Figure 6 is a plot of the RMS of the normalized residuals for the simulator test when there is no evil present. As expected, the sum of the squares of the residuals is quite small with the majority of the values less than 0.03%. This shows that the MEDLL algorithms are converging and the modeling is good. There is a slight increase in the sum squared residual value at 0.9 chip multipath delay. At that delay, the Multipath Meter's estimate of the actual delay is very good. As a result, the increased residual value can be correlated to the delay of the multipath when doing further analysis with evil waveforms.



Figure 6: Sum Squared correlator residuals, $a_m = 0.5$

The scale of Figure 6 may seem too large, but it has been chosen for easier comparisons to the following results shown in Figures 16, 21, and 26.

DETECTION USING MDE AND MDR VALUES

As previously stated, detection of anomalous satellite signals can be done by examining the minimum detectable error (MDE) of a single normalized correlator value (normalized by the punctual value) or by looking at the minimum detectable ratio (MDR) of correlator values across the peak of the correlation function.

In the presence of multipath, but not in the presence of evil waveforms, MDE and MDR values of the multipath corrected and uncorrected correlator values are shown in Figures 7 through 10.

MDR1 is the ratio of the tracking error between the correlators at ± 0.025 chips (tracking pair) to the correlators at ± 0.075 chips (non-tracking pair), normalized by the punctual correlation value. MDR2 is the ratio of the tracking error between the correlators at ± 0.025 (tracking pair) chips to the correlators at ± 0.125 chips (non-tracking pair), normalized by the punctual correlation value. The expected value for all of the MDR values is zero since we expect the correlation function to be symmetrical.

MDE1 is the correlator value at +0.025 chips normalized by the punctual correlator value. MDE2 is the correlator value at +0.125 chips normalized by the punctual correlator value. The expected value for the correlator at +0.025 chips would nominally be 0.975 but is shown in Figure 9 to be closer to 0.989. This is due to the rounding of the correlation peak from the receivers' RF filters. The expected value for the +0.125 correlator is 0.885, slightly higher due to rounding of the correlation function than the nominal value of 0.875.

With the MEDLL implementation, there are many more MDE and MDR values that can be checked. Only those mentioned above have been chosen in order to limit the size of this paper.



Corrected and Unconnected MOR # 025-8.0751 in the processory



Figure 7: MDR1 (0.025-0.075), No Evil



Figure 8: MDR2 (0.025-0.125), No Evil



Figure 9: MDE1 (+0.025), No Evil

From Figures 7 through 10, we can see that the corrected correlator values represented by the lightly shaded line (pink) are significantly more consistent that the uncorrected values represented by the dark shaded line (blue). By reducing the noise and impact of the multipath signal on the MDE and MDR value, the threshold for detection can be significantly lowered. This will improve the ability to detect evil waveforms when multipath is present thereby reducing the false alarm rate. Another advantage is that there is little need for smoothing of the correlator values to reduce the influence of multipath, since the multipath effects have been removed by the MEDLL. This is turn will reduce the time to alarm for detecting evil waveforms.



Figure 10: MDE2 (+0.125), No Evil

What is of most interest now is how the multipath corrected correlator values behave in the presence of evil waveforms, and the ability of the Multipath Meter to detect their presence even with the multipath corrected correlator values.

ANALYSIS OF THREAT MODEL A IN THE PRESENCE OF MULTIPATH

Threat model A results are shown for a maximum Δ value of 0.12, with $a_m = 0.5$. Figures 11 through 14 show the multipath corrected and uncorrected MDE and MDR values. An additional solid black line has been added to the figures to show the metric value under the no-fault condition (no multipath and no satellite failure) that corresponds to the expected values stated in the previous section.

With the uncorrected correlator values, we can see that there are certain multipath conditions that would render the detection of the evil waveform impossible when using uncorrected correlator values. The MDE and MDR metric cross the no-fault line numerous times for the uncorrected metrics. For the multipath corrected metrics, we can see that from Figure 14 that no multipath will ever create this undetectable condition on the MEDLL since the metric is always above the no fault line. Furthermore, Figure 14 seems to show better detectability results when compared to Figure 13 seeing as the metric crosses the no-fault line at short delay multipath in Figure 13 and not in Figure 14. This indicates that the correlators that are further away from the peak are better suited for MDE type fault detection since multipath effects can be better removed, for threat model A type failures.



Figure 11: MDR1 (0.025-0.075), Threat Model A



Figure 12: MDR2 (0.025-0.125), Threat Model A



Figure 13: MDE1 (+0.025), Threat Model A

MDR values, as seen in Figures 11 and 12 seem to have difficulty detecting threat model A type failures when very short delay multipath or very long delay multipath are present. This is evident in the oscillations of the metric value around the no-fault line when short delay or long delay multipath is present.



Figure 14: MDE2 (+0.125), Threat Model A

We can also examine the D/U plot for threat model A in the presence of multipath. Figure 15 shows these results.

We can see that threat model A type failures result in the estimation of a very strong multipath signal when in fact there is no multipath present. This condition is shown as the first data point on the graph when there is a multipath delay of zero.

We can also see from Figure 15 that there are some multipath conditions with threat model A that produce a secondary peak which is more powerful that the direct path signal! As stated previously, it is always assumed that any multipath signal is of lower power than the direct path. With evil waveforms, this is no longer the case. We can see this by the estimate of a negative D/U between 0.1 and 0.2-chip multipath delay. Should a receiver lock onto this signal instead of the direct path, hazardous misleading information would most certainly be output.



Figure 15: D/U for Maximum Δ Evil with Multipath

The sum-squared residuals from the estimation process also show that there is significant residual error after removing the estimate of the multipath. These residuals are seen in Figure 16. Comparing Figures 6 to 16, we can see the precise magnitude of the difference. Increased residual values from the estimation can be attributed to one of two things: 1) the inability of the MEDLL to properly estimate the multipath signal, or 2) the signal itself is shaped such that proper estimation is difficult. This difficulty can arise from either multiple multipath signals [9] or the presence of an evil waveform.



Figure 16: Sum squared correlator residuals, Threat A

ANALYSIS OF THREAT MODEL B IN THE PRESENCE OF MULTIPATH

The threat model B parameters used for testing were σ =0.8 and f_d=4. This was chosen because the low frequency waveform with little damping generates the most distortions in the correlation function. MDE and MDR results showing the uncorrected correlator values, the multipath corrected correlator values, and the no-fault line are in Figures 17 through 20.

We can see from the figures that again, there are certain multipath conditions that will cause the satellite failure to go undetected using standard, non-multipath corrected correlator measurements for MDE and MDR values. The combinations of the MDR values shown in Figures 17 and 18 along with the MDE values in Figure 19 will detect all occurrences of this threat model B failure, regardless of multipath on the signal. Results in Figure 20 show that the multipath corrected MDR value will not detect the failure when there is less than 0.2 chips of multipath. This only reinforces the need for multiple monitoring points on the correlation function using both MDE and MDR values in order to detect all failures in the threat space.

Residuals from the multipath estimation process further exemplify the presence of an anomalous signal. The residuals in Figure 21 are again significantly greater then those present in Figure 6.



Figure 17: MDR1 (0.025-0.075), Threat Model B



Figure 18: MDR2 (0.025-0.125), Threat Model B

Connected and Unconnected MDE (H0.015) in the presence of Helitipath and Threat B



Figure 19: MDE1 (+0.025), Threat Model B



Figure 20: MDE2 (+0.125), Threat Model B



Figure 21: Sum squared correlator residuals, Threat B

ANALYSIS OF THREAT MODEL C IN THE PRESENCE OF MULTIPATH

Threat model C has the potential to introduce the most distortion of any of the three models, since we are combining all three parameters of distortion. With Δ =0.12, σ =0.8, and f_d=7.3, the correlation function is quite devious and is as described in Figure 22. Increasing the complexity of the correlation function with multipath poses real problems for detection using non-multipath corrected correlator values.



Figure 22: Threat Model C correlation function

Figures 23 through 26 show the uncorrected, corrected, and no-fault lines for the tested MDE and MDR values. We can see from these figures that certain multipath scenarios even prove to be difficult to detect using the multipath corrected correlator values, especially with very short delay multipath. Figure 25 showing the very close in correlator at +0.025 chips indicates many crossings of the no-fault line.



Figure 23: MDR1 (0.025-0.075), Threat Model C

rected SEPR (8.425-8.735) in the presence of S



Figure 24: MDR2 (0.025-0.0125), Threat Model C



Figure 25: MDE1 (+0.025), Threat Model C



Figure 26: MDE2 (+0.125), Threat Model C

However, we also have additional parameters from the Multipath Meter that we can examine for satellite failures. If we look at the sum squared of the residual error from the estimation process, we can see that the residuals indicate that the MEDLL is not estimating the multipath very well. This is the case for instances of evil waveforms, which has also been shown in Figures 16 and 21. For the threat model C testing, the residuals from the MEDLL can be seen in Figure 27. Comparing the residuals from Figure 6 to Figure 27, we can see that there is a significant change in the magnitude of the residuals over all multipath delays.



Figure 26: MEDLL Residuals, Threat Model C

CONCLUSIONS

The results from this analysis show that the Multipath Meter is useful for signal quality monitoring. For SQM the D/U, residuals, and multipath corrected correlator values can be used in harmony with one another to detect satellite signal failures. Using this approach provides for fewer false alarms due to multipath, since the multipath effects have been largely removed, and a shorter time to alarm, since the multipath corrected correlator measurements are less noisy. The D/U estimate can also be used to detect hazardous conditions when there is a secondary peak that is more powerful that the direct path signal.

The detection of the evil waveforms was shown to be possible even when multipath corrected correlator values were used with no multipath present.

When the MEDLL is unable to adequately estimate and remove multipath effects (as seen in the large residual values), the pseudorange accuracy shown in Figure 2 cannot be assured. Regardless of the source of the estimation error, either from signal failure or multiple multipath signals [9], the satellite measurements should not be used when residual values are extremely large.

Further examination of additional points in the threat space are required to completely determine detection thresholds using this method. In addition, testing using live data with the 16 MHz MEDLL implementation is required.

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