Positioning Leadership

GPS Position Accuracy Measures

Overview

There are various statistical methods of describing specifications for GPS receiver and it can seem confusing or misleading the meaning of such statistics. The purpose of this document is to introduce some of the most commonly used accuracy measures, and their relationships to one another.

1 Accuracy and Precision

Accuracy and precision are often used to describe the how good is the position acquired by GPS receiver. A distinction should be made between accuracy and precision. Accuracy is the degree of closeness of an estimate to its true, but unknown value and the precision is the degree of closeness of observations to their means. Figure 1 illustrates various relationships between these two parameters. The true value is located at the intersection of the crosshairs, the center of the shaded area is the location of the mean estimate, and the radius of the shaded area is a measure of the uncertainty contained in the estimate.
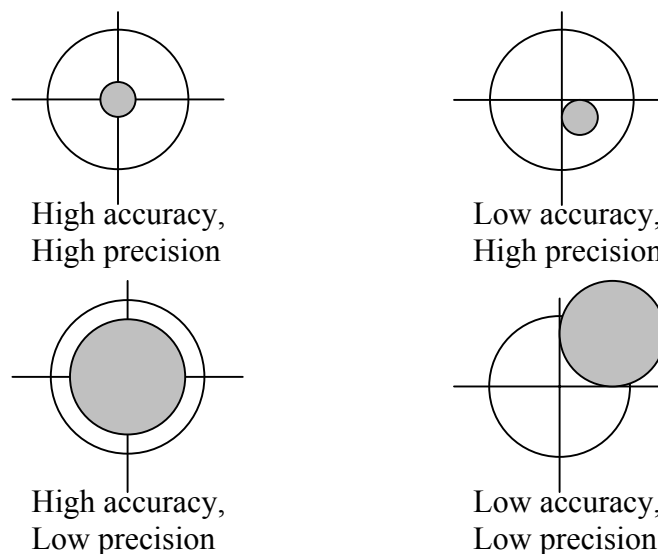

High accuracy,
High precision

Low accuracy,
High precision

High accuracy,
Low precision

Low accuracy,
Low precision

Figure 1: Accuracy versus Precision

2 Accuracy Measures - 2D

When GPS positions are logged over time, the positions are scattered over an area due to measurement errors. This dispersion of points is called a scatter plot, which GPS manufacturers use to characterize their equipment's accuracy. The area within which the measurements or estimated parameters are likely to be is called the confidence region. The confidence region is then analyzed to quantify the GPS performance statistically. The confidence region with a radius describes the probability that the solution will be within the specified accuracy.

Figure 2 shows the scatter plot of the L1 single point positions observed by the actual NovAtel OEM4-G2 receiver for 24 hours and its accuracy radii of CEP and DRMS.
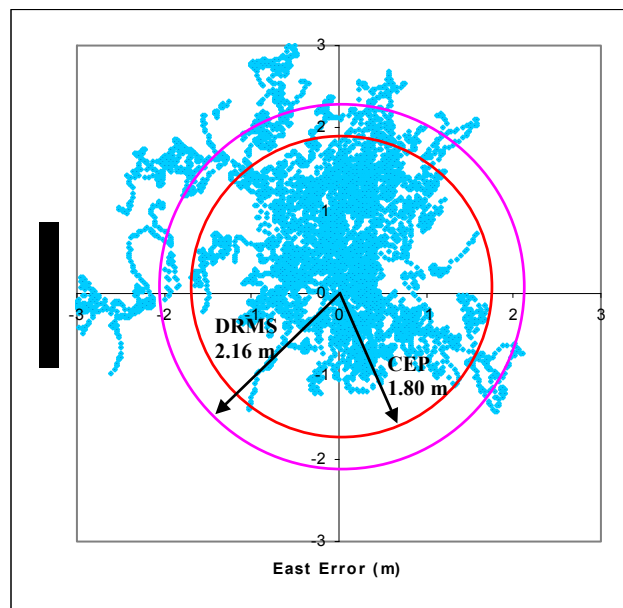

Figure 2: L1 Single Point positions collected on the rooftop of NovAtel building for 24 hours using NovAtel OEM4-G2 receiver and GPSAntenna Model 502.

2.1 Distance Root Mean Squared (DRMS)

DRMS is a single number that expresses 2D accuracy. In order to compute the DRMS of horizontal position errors, the standard errors (σ) from the known position in the directions of the coordinate axis are required.

DRMS is the square root of the average of the square errors which is defined as follows:

$$\text{DRMS} = \sqrt{\sigma_x^2 + \sigma_y^2}$$

❖ Standard errors (σ) of estimated coordinates (x, y) of each point being positioned can be predicted from the corresponding variances on the diagonal of the covariance matrix.

2.2 Circular Error Probability (CEP)

CEP refers to the radius of a circle in which 50% of the values occur, i.e. if a CEP of 5 meters is quoted then 50% of horizontal point positions should be within 5 meters of the true position. The radius of the 95% is often quoted and the term R95 used. R95 is CEP with the radius of the 95% probability circle.

Table 1 describes the most commonly used position accuracy measures and their probability.

Table 1: 2D Position Accuracy Measures

Accuracy Measures	Formula	Probability	Definition
DRMS	$\sqrt{\sigma_x^2 + \sigma_y^2}$	65%	The square root of the average of the squared horizontal position errors.
2DRMS	$2\sqrt{\sigma_x^2 + \sigma_y^2}$	95%	Twice the DRMS of the horizontal position errors.
CEP	$0.62\sigma_y + 0.56\sigma_x$ (Accurate when $\sigma_y / \sigma_x > 0.3$)	50%	The radius of circle centered at the true position, containing the position estimate with probability of 50%.
R95	$R(0.62\sigma_y + 0.56\sigma_x)$ (R=2.08, when $\sigma_y / \sigma_x = 1$)	95%	The radius of circle centered at the true position, containing the position estimate with probability of 95%.

2.3 Examples of 2D Accuracy Measures

Example 1:

Given standard deviations:

σ_x = 1.3 m
σ_y = 1.5 m

Then accuracy measures can be calculated as:

- CEP = $0.62 \times 1.3 + 0.56 \times 1.5 = 1.65$ m
- DRMS = $\sqrt{1.3^2 + 1.5^2} = 1.98$ m
- 2DRMS = $2\sqrt{1.3^2 + 1.5^2} = 3.96$ m
- 3DRMS = $3\sqrt{1.3^2 + 1.5^2} = 5.85$ m

Example 2:

How to convert one accuracy measure to another?

When 1.8 meter of CEP is given, what is the position accuracy for 2DRMS?

Use Table 2 to find equivalent multiplier for the conversion and follow the steps below.

1. Go down the "2DRMS" column to the "CEP" row.
2. The multiplier in this cell is 2.4
3. CEP = 1.8 x 2.4 = 4.32 m

Example 3:

Which is more accurate, 1.8 meters of CEP or 3 meters of RMS (3D)?

1. In Table 4, go down the "RMS (3D)" to the "CEP" row.
2. The multiplier in this cell is 2.5
3. RMS (3D) = CEP x 2.5
4. CEP = RMS (3D) / 2.5 = 3 / 2.5 = 1.2

The position accuracy with 3 meters of RMS (3D) will be 1.25 meters of CEP, therefore 3 meters of RMS (3D) is more accurate than 1.8 meters of CEP.

Table 2 gives the multipliers required to go from one accuracy measure to another assuming that $\sigma_x / \sigma_y = 1$ and that VDOP / HDOP = 1.9 and PDOP / HDOP = 2.1.

Table 2: Equivalent Accuracy Multipliers

RMS (Vertical)	CEP	RMS (Horizontal)	R95 (Horizontal)	2DRMS	RMS (3D)	SEP	
1	0.44	0.53	0.91	1.1	1.1	0.88	RMS (Vertical)
	1	1.2	2.1	2.4	2.5	2.0	CEP
		1	1.7	2	2.1	1.7	RMS (Horizontal)
			1	1.2	1.2	0.96	R95
				1	1.1	0.85	2DRMS
					1	0.79	RMS (3D)
						1	SEP

3 Accuracy Measures – 3D

Similar to 2D accuracy measures there are many representations of 3D accuracy with various probabilities. 3D accuracy measures are conceptually similar to those in 2D expanded by one dimension, the vertical accuracy. Spherical Error Probable (SEP) corresponds to CEP in 2D, while Mean Radial Spherical Error (MRSE) corresponds to DRMS in 2D.

Table 3 lists various methods for position accuracy measures in 3D and their definitions.

Table 3: 3D Position Accuracy Measures

Accuracy Measures	Formula	Probability	Definition
SEP (Spherical Error Probable)	$0.51(\sigma_y + \sigma_x + \sigma_z)$	50%	The radius of sphere centered at the true position, containing the position estimate in 3D with probability of 50%
MRSE (Mean Radial Spherical Error)	$\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$	61%	The radius of sphere centered at the true position, containing the position estimate in 3D with probability of 61%
90% Spherical Accuracy Standard	$0.833(\sigma_y + \sigma_x + \sigma_z)$	90%	The radius of sphere centered at the true position, containing the position estimate in 3D with probability of 90%
99% Spherical Accuracy Standard	$1.122(\sigma_y + \sigma_x + \sigma_z)$	99%	The radius of sphere centered at the true position, containing the position estimate in 3D with probability of 99%

4 Dilution of Precision (DOP)

The DOPs provide a simple characterization of the user-satellite geometry. DOP is related to the volume formed by the intersection points of the user-satellite vectors, with the unit sphere centered on user. Larger volumes give smaller DOPs. Lower DOP values generally represent better position accuracy. The role of DOP in GPS positioning, however, is often misunderstood. A lower DOP value does not automatically mean a low position error. The quality of a GPS-derived position estimate depends upon both the measurement geometry as represented by DOP values, and range errors caused by signal strength, ionospheric effects, multipath etc.

5 Final Points

If you require any further information regarding the topics covered within this application, please contact:

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